Prediction of Pressure Distribution Change with Time for Non-Newtonian Fluids in Porous Media Using Numerical Simulation

Muhammad Obeidat
Department of Oil and Gas Engineering
Azerbaijan State Oil and Industry University
Baku, Azerbaijan
muhob1207@gmail.com

***Abstract*—The increasing demand for hydrocarbons and the depletion of conventional oil fields have increased the need for development of unconventional oil reserves, which include non-Newtonian oil fields. That is why it is important to understand the physical properties of non-Newtonian fluids and study their behavior in a reservoir. The focus of this study is numerical simulation of the flow of non-Newtonian Herschel-Bulkley fluids in porous media under unsteady-state conditions the purpose of which is to predict the change of pressure distribution in an oil reservoir with time. Firstly, a modified diffusivity equation for Herschel-Bulkley fluids is derived. Then the proposed numerical method is validated by comparing numerical simulation results to exact analytical solutions for simpler fluid models. Then numerical simulation is done for complex fluid models and the results demonstrate the impact of such non-Newtonian parameters as power-law index and initial pressure gradient on the pressure distribution in the reservoir. It is concluded that the impact of power-law index is very strong, while for the initial pressure gradient it is not significant.**

Keywords*—*unconventional oil reserves, non-Newtonian fluids, Herschel-Bulkley model, fluid flow in porous media, numerical simulation

I. INTRODUCTION

Due to the continuously increasing demand for hydrocarbons, the production of hydrocarbons has also increased in the recent years. The continuous exploitation of conventional oil fields has led to their depletion, and therefore, to the decrease of production from those fields. That is why nowadays interest in developing unconventional oil fields is growing, as they account for approximately 70% of total world oil reserves [1, 2]. The research of physical properties of unconventional oil fields and development of new methods of their extraction would be highly beneficial since it would allow to fulfill the growing energy demand in the future [3].

Unconventional hydrocarbon reserves include but are not limited to non-newtonian oil fields, gas hydrates, light tight oil, shale oil and tar sands [4]. For a long time development of such reserves has not been common which was due to several factors which include economic unprofitability and presence of much more profitable conventional oil reserves as an alternative [5, 6]. However, new technologies like Cyclic Steam Stimulation (CSS) and Steam-assisted Gravity Drainage (SAGD) have allowed to start profitable exploitation of many hydrocarbon reserves the development of which was previously considered economically not viable [7]. The focus of this work is numerical simulation of flow of non-Newtonian fluids in porous media under unsteady-state conditions.

II. PROBLEM STATEMENT

The key differences between non-Newtonian and Newtonian fluids are written below:

- Non-Newtonian fluids are characterized by the presence of yield stress. The flow condition for such fluids is that the shear stress that occurs in the fluid needs to become higher than the yield stress, or otherwise, the fluid will not flow [8].
- Unlike Newtonian fluids, for which the relationship between shear stress and shear rate is strictly linear, for non-Newtonian fluids this is not the case [9].
- For non-Newtonian fluids, the dynamic viscosity coefficient is replaced with consistency index, which is measured in different units which depend on the power-law index.

Many mathematical models that describe the flow of non-Newtonian fluids exist, however, the most general one is known as the Herschel-Bulkley model, and it is defined as [10]:

$$\tau = \tau_0 + K\dot{\gamma}^n, \tag{1}$$

where τ_0 is the yield stress, $\dot{\gamma}$ is the shear rate, K is the consistency index and n is the power-law index. For Newtonian fluids τ_0 would be equal to zero and n would be equal to 1, while K would be equal to the dynamic viscosity μ. The most commonly used model for describing non-Newtonian fluid flow in porous media that is based on the Herschel-Bulkley model and aligns well with the experiments is defined as follows [11]:

$$v = \left(\frac{k}{\mu_{ef}}\left(-\frac{\partial P}{\partial x} - G_0\right)\right)^{\frac{1}{n}}, \tag{2}$$

where k is the porous rock permeability, $\frac{\partial P}{\partial x}$ is the pressure gradient, G_0 is the initial pressure gradient that needs to be exceeded in order for the fluid to start flowing in porous media, and μ_{ef} is the effective viscosity. For Newtonian fluids n becomes 1 and G_0 becomes 0, and the formula (2) reduces to the classic Darcy law. Note that in the original work there is no minus sign before ∂P/∂x in equation above which is a typo, and it has been corrected here.

Many formulas (empirical, analytical and semi-empirical) have been proposed in the literature for the determination of the initial pressure gradient G_0. For instance, the experiments conducted in the work [12] have shown that for sands the initial pressure gradient can be determined as:

$$G_0 = \frac{0.052\tau_0}{k^{0.62}}, \qquad (3)$$

and for carbonate rocks it can be determined as:

$$G_0 = \frac{0.003\tau_0}{k^{1.596}}. \qquad (4)$$

The effective viscosity μ_{ef} is determined from the formula [11]:

$$\mu_{ef} = \frac{K}{12}\left(9 + \frac{3}{n}\right)^n (150k\phi)^{\frac{1-n}{2}}. \qquad (5)$$

The formula (5) is derived from the semi-analytical modification of the Blake-Kozeny equation for non-Newtonian fluids. ϕ stands for the porosity of the porous rock. The purpose of this work is to use the finite difference method of numerical simulation, a certain set of boundary conditions and the formula (2) in order to describe the non-Newtonian Herschel-Bulkley fluid flow in porous media under unsteady-state conditions, meaning that we are aiming to find the pressure distribution in an oil reservoir and how this distribution changes with time. For that, we first need to derive an analogue of oil diffusivity equation but for the Herschel-Bulkley fluids.

III. Numerical simulation

A. Derivation of the modified diffusivity equation for Herschel-Bulkley fluids flowing in porous media

The continuity equation has the following form in cylindrical coordinates in porous media [13]:

$$\frac{\rho v_r}{r} + \frac{\partial(\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\partial(\phi\rho)}{\partial t} = 0. \qquad (6)$$

Here ρ is the density of the fluid, r is the radial coordinate, θ is the azimuth, and z is the vertical coordinate. v_θ, v_r, v_z stand for the corresponding velocities in the directions of the mentioned coordinates.

We assume that the reservoir has a circular form of radius r_e. In the center of the reservoir a single production well of radius r_w is located. Oil is produced due to its own elastic expansion energy. We also assume that there is no fluid filtration in the directions θ and z. In that case the continuity equation becomes:

$$\frac{\rho v_r}{r} + \frac{\partial(\rho v_r)}{\partial r} + \phi\frac{\partial \rho}{\partial t} + \rho\frac{\partial \phi}{\partial t} = 0. \qquad (7)$$

In the work [14] it is mentioned that the relationship between the rock porosity and normal stress σ is generally non-linear, however, in the interval from σ=0 to 10 MPa, this relationship can be considered linear and can be expressed as:

$$\phi = \phi_0 - \beta_c(\sigma - \sigma_0), \qquad (8)$$

where β_c stands for the porous media compressibility, σ_0 is some reference stress, and ϕ_0 is a known porosity at the reference normal stress. The relationship between the overburden pressure P_v, reservoir pressure P and normal stress σ can be expressed as [14]:

$$P_v = \sigma + P. \qquad (9)$$

Since the overburden pressure does not change with time, we differentiate the equation (9) by time and get the following:

$$\frac{\partial \sigma}{\partial t} = -\frac{\partial P}{\partial t} \qquad (10)$$

By taking into account equations (8) and (10), we get that:

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial \sigma}\frac{\partial \sigma}{\partial t} = -\beta_c\frac{\partial \sigma}{\partial t} = \beta_c\frac{\partial P}{\partial t}. \qquad (11)$$

The density of the fluid filtering in the reservoir, to a first approximation, linearly depends on the pressure [14]:

$$\rho = \rho_0\left[1 + \beta_f(P - P_0)\right], \qquad (12)$$

where β_f is the fluid compressibility, P_0 is some reference pressure and ρ_0 is a known density at the reference pressure. By differentiating by time, we then get the following:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial P}\frac{\partial P}{\partial t} = \rho_0\beta_f\frac{\partial P}{\partial t}. \qquad (13)$$

Now we insert equations (11) and (13) into equation (7) and get the following:

$$\frac{\rho v_r}{r} + \frac{\partial(\rho v_r)}{\partial r} + \phi\rho_0\beta_f\frac{\partial P}{\partial t} + \rho\beta_c\frac{\partial P}{\partial t} = 0. \qquad (14)$$

Considering that the compressibility of fluids is very low, we can assume that $\rho \approx \rho_0$. In this case equation (14) becomes:

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \beta\frac{\partial P}{\partial t} = 0, \qquad (15)$$

where β is defined as:

$$\beta = \beta_c + \phi\beta_f. \qquad (16)$$

On the other hand, v_r can be expressed through pressure using equation (2). After this substitution, the equation (15) becomes:

$$\frac{1}{n}\left(\frac{\partial P}{\partial r} - G_0\right)^{\frac{1}{n}-1}\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\left(\frac{\partial P}{\partial r} - G_0\right)^{\frac{1}{n}} = \frac{1}{\chi}\frac{\partial P}{\partial t}, \qquad (17)$$

where χ is defined as:

$$\chi = \frac{1}{\beta}\left(\frac{k}{\mu_{ef}}\right)^{\frac{1}{n}}. \qquad (18)$$

Equation (17) is the general modified diffusivity equation for non-Newtonian fluids. Obviously, in case if we take G_0 = 0, and n = 1, then equation (17) easily reduces to:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\chi}\frac{\partial P}{\partial t}, \quad (19)$$

which is the well-known diffusivity equation for Newtonian fluids [15]. Let's assume that the boundary conditions are:

- P(r, t=0) = P_e. Pressure is equal to initial pressure at any point in the reservoir at t = 0.
- P($r \to \infty$, t) = P_e. Very far from the production well the pressure always remains equal to the initial pressure.
- $\left(r\frac{\partial P}{\partial r}\right)_{r=0,t} = \frac{Q_0\mu}{2\pi kh}$. At r = 0, the expression $r\frac{\partial P}{\partial r}$ is considered constant. Q_0 corresponds to the constant flow rate of the production well.

In this case the general analytical solution of the equation (19) looks as follows [16]:

$$P(r,t) = P_e - \frac{Q_0\mu}{4\pi kh}\left[-Ei\left(-\frac{r^2}{4\chi t}\right)\right]. \quad (20)$$

B. Numerical simulation method – description and validation

In the work [17] a similar analytical solution has been derived for Power Law fluids flowing in porous media. Similarly, in the work [18], an analytical solution for pressure distribution has been derived for Bingham fluids flowing in porous media. No analytical solution is present for Herschel-Bulkley fluids which is explained by the big mathematical complexity of the Herschel-Bulkley model. We used the aforementioned exact analytical solutions in order to validate the used numerical method (ensuring same fluid model and same boundary conditions). The results were very close and that is why the chosen numerical simulation method has been used to simulate the behavior of Herschel-Bulkley fluids for which no analytical solution exists. Results of comparing chosen numerical simulation method with analytical solution (20) are demonstrated on Fig. 1 below.

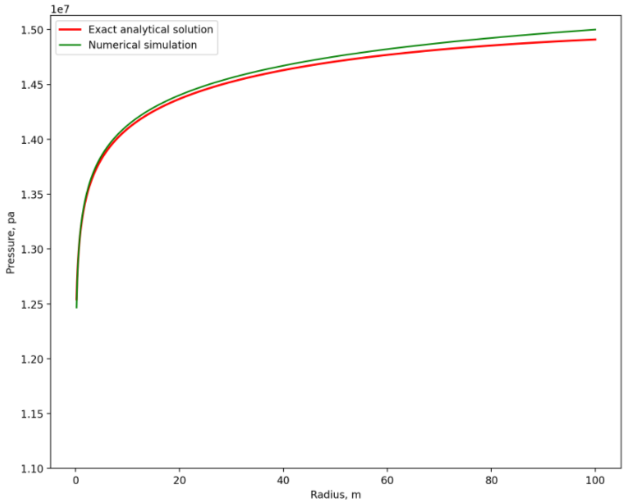

Fig. 1. Comparison of numerical results to analytical solution

Equation (17) can be expressed in the following way using the finite difference method [19]:

$$P_r^{t+1} = P_r^t + \chi\Delta t\left[\frac{1}{n}\left(\frac{P_r^t - P_{r-1}^t}{\Delta r} - G_0\right)^{\frac{1}{n}-1}\frac{P_{r+1}^t - 2P_r^t + P_{r-1}^t}{\Delta r^2} + \frac{1}{r}\left(\frac{P_r^t - P_{r-1}^t}{\Delta r} - G_0\right)^{\frac{1}{n}}\right]. \quad (21)$$

We use the following boundary conditions:

- $P(r_w, t) = P_0^t = P_w$. Meaning that we consider the pressure at the oil well wall to be constant and equal to P_w.
- $P(r_e, t) = P_m^t = P_e$. Meaning that we consider the pressure at the outer boundary of the reservoir to be constant and equal to P_e.
- $P(r, 0) = P_r^0 = P_e$. In all points of the reservoir except the well wall at the initial time moment, the pressure is considered to be equal to P_e. At the wall well at the initial time moment the pressure becomes equal to P_w what creates the pressure gradient and the flow.

C. Numerical simulation results

Fig. 2 shows the results of numerical simulation - the relationship between reservoir pressure, radius and time in 3D. The parameter values that were used in this case are given in the Table 1. In this case we are using a classic example of Herschel-Bulkley fluids, as the initial pressure gradient is considered to be $G_0 = 10000\ Pa/m$, and the power-law index is taken equal to n = 1.2. It can be seen that pressure stays constant in the wellbore. With the increase of distance from the wellbore the pressure also increases. The most abrupt increase of pressure can be noticed in areas close to the wellbore. We can also observe how the pressure decreases with time in all parts of the reservoir. It is important to emphasize that generally speaking, this relationship strongly depends on the chosen boundary conditions. The vertical orange surface shows how the position of the pressure penetration front changes with time. It can be seen that the linear speed of the pressure penetration front decreases with the increase of penetration.

Fig. 3 shows the pressure distribution in the reservoir at t = 10000 seconds for various values of power-law index n. The significant impact of this parameter can be observed – we can clearly see that the decrease of n strongly decreases the pressure at every point of the reservoir, especially in areas that are close to the well. The vertical lines demonstrate the current position of the pressure penetration front for each corresponding power-law index value.

The decrease of power-law index also increases the position of the pressure penetration front. This is a very important parameter, especially in reservoirs with circular shape like in this case. The reason for this is that when a pressure drop starts in a certain area of a reservoir, it means that oil starts being produced from there and the oil recovery factor from that specific area starts increasing. If we take for instance an oil recovery factor of 1%, then due to the geometry of a circular reservoir, an area having this oil recovery factor that is far away from the well (area located at a certain distance r from the well) would lead to much higher total oil production than an area having the same oil recovery factor that is close to the well.

Fig. 4 shows the pressure distribution in the reservoir at t = 10000 seconds for various values of initial pressure gradient G_0. The impact of this parameter is not so significant compared to the impact of the power-law index. In areas closer to the wellbore an increase of G_0 leads to lower pressure for the same time value, while in areas far from the wellbore, a decrease of G_0 leads to lower pressure. The decrease of G_0 obviously leads to higher penetration of the pressure change front into the reservoir. This means that higher initial pressure gradients lead to higher oil recovery factor in areas of the reservoir that are closer to the well, but to lower recovery factors in areas that are far from the well. The analysis shows that generally, the decrease of the initial pressure gradient has a positive impact since it increases the total oil production from the reservoir, which can be explained by the impact of circular reservoir geometry on the total oil production.

Table 1. Values of used parameters

Parameter name	Parameter value
k, m^2	$0.1 * 10^{-12}$
h, m	20
ϕ_0	0.1
P_w, Pa	5000000
P_e, Pa	15000000
$G_0, Pa/m$	10000
$K, Pa * s^n$	$5 * 10^{-3}$
$Q_0, m^3/s$	0.001
β_f, Pa^{-1}	$0.4 * 10^{-9}$
β_c, Pa^{-1}	$0.2 * 10^{-9}$
$\Delta t, s$	0.05
$\Delta r, m$	0.1
r_w, m	0.2
r_e, m	100
t_{max}, s	50000
n	1.2
$\rho_0, kg/m^3$	850

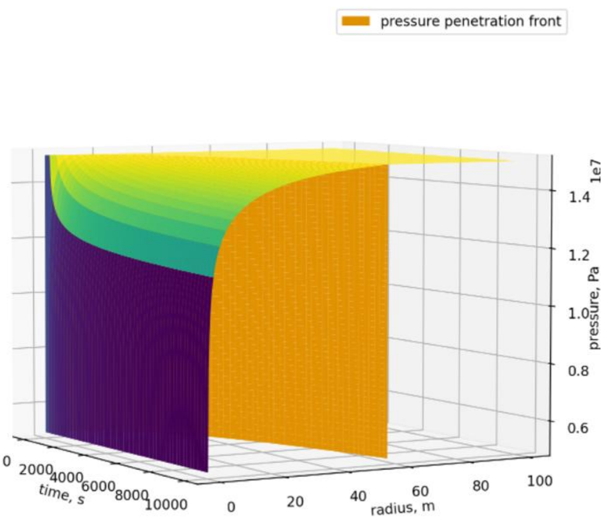

Fig. 2. Relationship between pressure, radius and time in 3D – results of numerical simulation

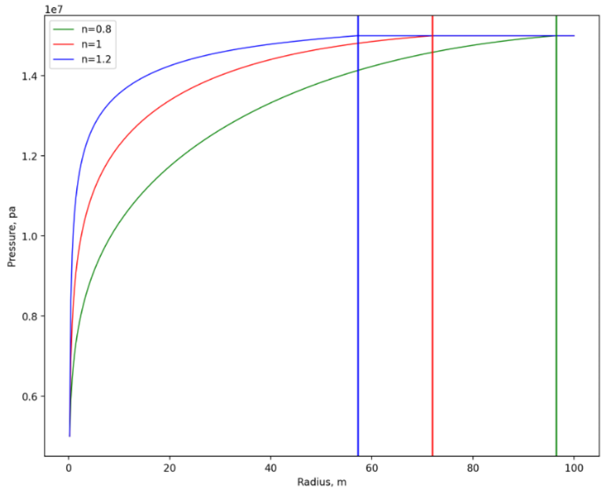

Fig. 3. Pressure distribution in the reservoir for various values of power-law index – results of numerical simulation

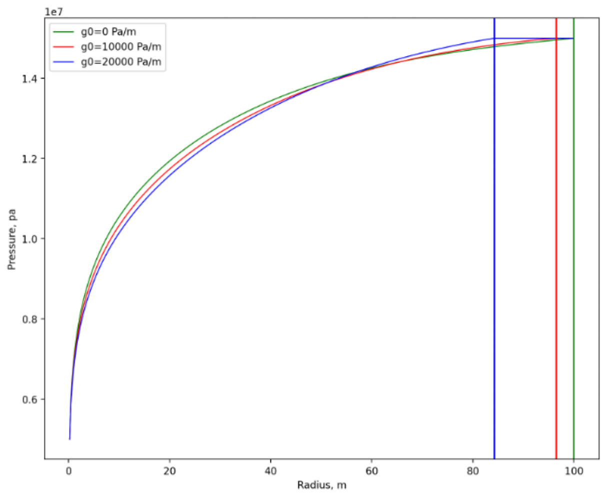

Fig. 4. Pressure distribution in the reservoir for various values of initial pressure gradient – results of numerical simulation

IV. Conclusions

The purpose of present work was to study the behavior of non-Newtonian Herschel-Bulkley fluids in porous media under unsteady state conditions – when pressure does not remain constant with time. To achieve this, a numerical method has been proposed and used, which has been validated by comparing the numerical simulation results with results of exact analytical solutions derived in previous literature for simpler fluid models. After that, numerical simulation was used in order to predict the pressure distribution in the reservoir at different time intervals for Herschel-Bulkley fluids.

The change of pressure penetration front in the reservoir with time has also been shown. Various pressure distributions were also obtained for different values of non-Newtonian fluid parameters which are the power-law index n and the initial pressure gradient G_0. It has been concluded that the impact of the power-law index n on the pressure distribution is very significant while the impact of G_0 is quite small.

Finally, it is important to emphasize that the impact of the specified parameters on the reservoir pressure depends on the chosen boundary conditions. For instance, if we assume that the flow rate at the wellbore stays constant while the pressure decreases with time, then the impact of the initial pressure gradient and power-law index on the pressure will not be exactly the same as shown on the graphs above.

REFERENCES

[1] Youssef Yatimi (2024). Advancement in heavy oil upgrading and sustainable exploration emerging technologies. Arabian Journal of Chemistry.

[2] Muther, T., Qureshi, H. A., Syed, F. I., Aziz, H., Siyal, A., Dahaghi, A. K., & Negahban, S. (2021). Unconventional hydrocarbon resources: geological statistics, petrophysical characterization, and field development strategies. Journal of Petroleum Exploration and Production Technology.

[3] Caineng Z. (2013). Unconventional hydrocarbon geology, 2nd edn. Beinjing: Geological Publishing House.

[4] Cander, H. (2012). Abstract: What Are Unconventional Resources? A Simple Definition Using Viscosity and Permeability. AAPG - Poster Presentation Annual Convention and Exhibition.

[5] Anon. (2012). Leader: America's oil bonanza. The Economist Newspaper Limited. The Economist.

[6] Song Y, L. Z. (2015). The concept and the accumulation characteristics of unconventional hydrocarbon resources. Pet Sci 12(4):563–572.

[7] Law, D. (2016). A New Heavy Oil Recovery Technology to Maximize Performance and to Maximize Performance and Minimize Environmental Impact. SPE Distinguished Lecturer Program. Society of Petroleum Engineers.

[8] Coussot, P. (2017). Bingham's heritage. Rheologica Acta.

[9] Pierre, S. (2016). Complex fluids: Modeling and Algorithms. Cham, Switzerland: Springer International Publishing.

[10] Herschel, W., Bulkley, R. (1926). Konsistenzmessungen von Gummi-Benzollösungen. Kolloid Zeitschrift, 39 (4): 291–300.

[11] Pascal, H. (1984). Dynamics of moving interface in porous media for power law fluids with yield stress . Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 211.

[12] Gafarov, Sh. (2005). Investigation of filtration parameters of non-Newtonian oil during flow in carbonate porous media [Исследование фильтрационных параметров неньютоновской нефти при течении в карбонатных пористых средах]. Neftegazovoe delo.

[13] Türkoğlu, P. D. (2018). ME 304 Fluid Mechanics II. Spring.

[14] Zheltov, Y.P. (1986). Oil fields development [Разработка нефтяных месторождений]. Moskva: Nedra.

[15] Fanchi, J. R. (2002). Shared Earth Modeling. Elsevier.

[16] Kushtanova, G.G. (2010). Subsurface hydromechanics [Подземная гидромеханика]. Faculty of Physics, Kazan (Volga Region) Federal University.

[17] H. PASCAL, F. P. (1985). Flow of non-newtonian fluid through porous media. Department of Physics, University of Alberta, Edmonton, Alberta, T6G 2J1, Canada .

[18] Wu, Y.S. (1990). Flow and Displacement of Bingham Non-Newtonian Fluids in Porous Media. SPE.

[19] Samarskii A.A., N. E. (1978). Methods for solving grid equations [Методы решения сеточных уравнений]. Moskva: Nauka.