Controlling the Rod Heating Process Using Feedback on Special Classes of Controls

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***Abstract* — The study proposes a numerical solution to feedback control problems for distributed parameter objects using the rod heating example. The approach involves observing the object's phase state at specific locations given inaccurate information on the object's initial state. The heat source's power serves as a control parameter in the considered problem. The notion of zonality is introduced, and a special class of zonal controls for the control parameter's values is defined. Zonality in this context means the constancy of the control parameter's values in each subset of the phase space determined by the set of values of the object's state measured at the observation points. Unlike the well-known approaches to synthesizing current control values, in which they depend on the value of the object's state, in the proposed approach, the values of the control parameters change only when the phase state transitions from one set of states to another. To determine the zonal controls, the original feedback control problem is reduced to a parametric optimal control problem that aims to optimize a finite-dimensional vector. Optimality conditions for the zonal control's values, which include formulas for the gradient of the objective functional with respect to the optimizable parameters, are derived.**

Keywords* — *zonal feedback control; distributed parameters object; heat conduction process; parabolic equation; gradient of functional.

I. Introduction

In recent decades, important research has been carried out in controlling industrial processes and technical objects, and corresponding devices have been developed. With the development of computing and measuring tools, feedback control systems for objects with lumped parameters have found wide applications in automatic control systems for various technical and industrial objects. The range of control problems involving feedback organization with objects is quite broad. At the same time, in recent decades, active research has been conducted to extend existing approaches and methods for systems with lumped parameters to control objects with distributed parameters, including feedback. The theory of distributed parameter control systems is essential in modern automatic control theory. Due to the characteristics of distributed objects, the synthesis of distributed control systems is, in most cases, more complex than a synthesis of lumped systems. Diffusion and heat conduction processes, many chemical-technological processes, elastic structures movement processes, as well as hydrodynamic, aerodynamic, and radiation processes, are all examples of distributed control objects. There is no formalized methodological approach to solving the problem of controlling objects with distributed parameters; therefore, it is necessary to use non-standard approaches and decision-making in each case. Many fundamental results have contributed to developing the theory of distributed parameter control systems through the works of Bryson A.E. and Yu-Chi Ho. [1], Lyons J.L. [2], Lurie K.A. [3], Fursikov A.V. [4], etc. Feedback control systems have been widely used in practical applications thanks to modern measuring and computing technology that enables large amounts of measuring and computation to be carried out in real time. This, in turn, has enabled the automation of processes in many industries, from manufacturing to agriculture. It has also allowed for more efficient use of resources and improved safety. Currently, automatic control and regulation systems are being developed for objects with distributed parameters using various principles, computational methods, and technical remote control (see, for example, [5, 6]).

The study is dedicated to the numerical solution of a control synthesis problem as applied to distributed parameter objects using special control action classes. It introduces the concept of "zonality" for synthesizable controls, meaning their value remains constant across subsets of the object's possible states. In contrast to known methods for synthesizing current values of control actions, where they depend on the state of the object, in the proposed approach, the values of control parameters change only when the phase state transitions from one set of states to another. The control actions' values are influenced by the type of feedback used and the functional relationship between the control and the object's currently observable state. In particular, the case of discrete feedback, in which the object's phase state is observed discretely at its predefined points, is examined. The control of many technical objects and technological processes is an example of the practical application of such problems. Each observation or feedback requires taking specific measures, so there is a cost associated with the materials and time involved in continuously monitoring these objects. The constancy of zonal control parameters determines the control system's robustness. It also ensures the enhancement of the technical performance of the equipment used within the control loop and the feasibility of synthesizable control actions with high accuracy. Optimal control of production and injection wells' flow rates and placement in an oil field when operating oil reservoirs under water-driven piston displacement mode [7], identification of the hydraulic resistance coefficient of the pipeline's linear segment under non-stationary flow of viscous fluids [8], as well as feedback control and identification problems as applied to lumped parameters objects [9 – 14] are just some examples of specific nonlinear inverse and optimization problems successfully solved numerically using the principle of zonality for control actions.

II. Problem Statement

As an example of the proposed approach, the problem of controlling the rod heating process in a furnace is studied. The following parabolic partial differential equation describes this process:

$$\frac{\partial \mathcal{T}(x,t)}{\partial t} = a\frac{\partial^2 \mathcal{T}(x,t)}{\partial x^2} + b[u(t) - \mathcal{T}(x,t)], \quad (1)$$

where $x \in (0,1)$, $t \in (0,\tau]$, $\mathcal{T}(x,t)$ is the rod's temperature at the point x at the moment of time t; $u(t)$ the power of the heat source, whose values depend on the hot air's temperature fed into the furnace; b the heat transfer coefficient; a the thermal diffusivity coefficient. Assume the following initial and boundary conditions, respectively:

$$\mathcal{T}(x,0) = \mathcal{T}_0 = \text{const}, \ \ x \in [0,1], \quad (2)$$

$$\frac{\partial \mathcal{T}(0,t)}{\partial x} = \frac{\partial \mathcal{T}(1,t)}{\partial x} = 0, \ \ t \in [0,\tau]. \quad (3)$$

Note that the exact value of the rod's temperature $\mathcal{T}_0$ at the initial moment is not known, but the set T of possible initial temperature values is specified along with the density function $\rho_{\mathrm{T}}(\mathcal{T}_0)$ that satisfies the following relations:

$$\int_{\mathrm{T}} \rho_{\mathrm{T}}(\mathcal{T}_0) d\mathcal{T}_0 = 1, \ \ \rho_{\mathrm{T}}(\mathcal{T}_0) \geq 0.$$

The function $\mathcal{T}(x,t) \in \mathrm{H}^{2,1}([0,1] \times [0,\tau])$ is a generalized unique solution of the problem (1) – (3) (see, for example, [2, 15]); namely, for an arbitrary function $\psi(x,t) \in \mathrm{H}^{2,1}([0,1] \times [0,\tau])$ such that for $x \in (0,1)$ and $t \in [0,\tau]$, $\psi(x,\tau) = 0$ and $\partial\psi(0,t)/\partial x = \partial\psi(1,t)/\partial x = 0$, the following equality holds:

$$\int_0^1 dx \int_0^\tau \mathcal{T}(x,t)\left[a\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{\partial\psi(x,t)}{\partial t} - b\,\psi(x,t)\right] dt$$

$$= -\int_0^1 \psi(x,0)\mathcal{T}_0 dx - \int_0^1 dx \int_0^\tau b\,u(t)\,\psi(x,t) dt$$

The piecewise continuous function $u(t)$, which acts as a control parameter, determines the optimizable power of the heat source. Certain constraints must be imposed on the control values due to technological requirements:

$$U = \{u(t):\ \underline{u} \leq u(t) \leq \overline{u},\ t > 0\},$$

where U represents the set of admissible control values; $\underline{u}$ and $\overline{u}$ are the minimal and maximal values, respectively, the control can assume.

Consider several thermal sensors installed along the length of the rod at n locations with coordinates x_i, $i = 1,2,\ldots,n$. With these sensors, information on the state of the heating process at the locations x_i can be input to the control system in the form of the vector

$$\overline{\mathcal{T}}(t) = \left(\overline{\mathcal{T}}_1(t), \overline{\mathcal{T}}_2(t), \ldots, \overline{\mathcal{T}}_n(t)\right) =$$

$$= \left(\mathcal{T}(x_1,t), \mathcal{T}(x_2,t), \ldots, \mathcal{T}(x_n,t)\right).$$

At the locations x_i, the object's state is measured at the predefined discrete time moments $t_j \in [0,\tau]$, $j = 0,1,2,\ldots,m$, where $t_0 = 0$ and $t_m = \tau$. To control the temperature within the rod, a regulator is synthesized that adjusts the temperature $u(t)$ of the heat source to maintain the rod's temperature $\mathcal{T}(x,\tau)$ at the desired level $\mathcal{T}^*(x)$ using temperature measurements sampled at the points x_i of the rod. For the rod heating process, the feedback control problem involves computing admissible values of the control $u(t)$ as a function of the rod's temperature measured at the locations x_i, i.e.,

$$u(t) = u\left(\overline{\mathcal{T}}(t)\right),\ u(t) \in U,\ t \in [0,\tau],$$

that minimizes an objective function given in the form

$$F(u) = \int_{\mathrm{T}} \Phi(u,\mathcal{T}_0)\rho_T(\mathcal{T}_0) d\mathcal{T}_0, \quad (4)$$

$$\Phi(u,\mathcal{T}_0) = \int_0^1 w(x)[\mathcal{T}(x,\tau;u,\mathcal{T}_0) - \mathcal{T}^*(x)]^2 dx,$$

for the case of non-fixed initial conditions (3). Here $w(x)$ is the weight function giving more importance or influence to some points along the rod's length; $\mathcal{T}(x,\tau;u,\mathcal{T}_0)$ the solution to the boundary-value problem (1) – (3) at the final moment of time that corresponds to admissible control values and a particular initial condition $\mathcal{T}_0 \in \mathrm{T}$; the predefined function $\mathcal{T}^*(x)$ characterizes the desired temperature distribution at the final moment of time. Functional (4) expresses the quality of the control process on average over all possible initial states $\mathcal{T}_0$ and the specified boundary conditions. Let the rod's temperature satisfy the inequalities

$$\mathcal{T}_{\min} \leq \mathcal{T}(x,\tau;u,\mathcal{T}_0) \leq \mathcal{T}_{\max}$$

for all possible combinations of initial and boundary conditions and admissible control values. Let us divide the range of temperature values $[\mathcal{T}_{\min}, \mathcal{T}_{\max}]$ into a finite number of intervals by the points ξ_k, $k = 0,1,2,\ldots,r$, where $\xi_0 = \mathcal{T}_{\min}$ and $\xi_r = \mathcal{T}_{\max}$, and

$$[\mathcal{T}_{\min}, \mathcal{T}_{\max}] = \cup_{k=1}^{r}[\xi_{k-1}, \xi_k).$$

Let us introduce n-dimensional hypercubes (parallelepipeds) in the space $\overline{\mathcal{T}}(\cdot) \in \mathbb{R}^n$:

$$\mathrm{Z}_{i_1,i_2,\ldots,i_n} = \left\{\left(\overline{\mathcal{T}}_1,\ldots,\overline{\mathcal{T}}_n\right): \mathcal{T}_{i_{s-1}} \leq \mathcal{T}(x_s,t;u,\mathcal{T}_0) \leq \mathcal{T}_{i_s}\right\} \quad (5)$$

$$i_s \in \{1,2,\ldots,r\},\ s = 1,2,\ldots,n.$$

with their total number equal to r^n. The number of each such hypercube, which will henceforth be called a "zone", is identified by the n -dimensional multiindex $\mathrm{I} = (i_1, i_2, \ldots, i_n)$. Each zone has a constant control value associated with it:

$$u(t) = v_{(i_1,i_2,\ldots,i_n)} = v_{\mathrm{I}} = \text{const},\ \ t \in \left[t_j, t_{j+1}\right), \quad (6)$$

$$\overline{\mathcal{T}}(t_j) \in \mathrm{Z}_{i_1,i_2,\ldots,i_n} = \mathrm{Z}_{\mathrm{I}},\ j = 0,1,2,\ldots,m-1.$$

i.e., the control $u(t)$ takes a constant value for the duration $t \in \left[t_j, t_{j+1}\right)$ that depends on the number of zone (5) to which the object's state $\overline{\mathcal{T}}(t_j)$ belongs. In exceptional situations, it is agreed to use the zonal control value of that adjacent zone into which the trajectory has transitioned if the object's state at the measurement point t_j is at the border of any zones. There are as many different values of the heat source's power as there are phase parallelepipeds, i.e., r^n. For convenience, instead of using multi-indices to identify the zonal control values, they are numbered sequentially as $v_1, v_2, \ldots, v_{r^n}$.

While controlling the rod heating process, the heat source's power given in the form of (6) changes only when the object's phase state transitions from one zone (5) to another. Thus, to solve the posed feedback control problem, the r^n-dimensional vector $v \in \mathbb{R}^{r^n}$ needs to be optimized. The resulting problem (1) – (6) on controlling the distributed parameters system is an example of a parametric optimal control problem. It has some specific features: first of all, there are no prescribed initial conditions; secondly, the optimizable control vector is finite-dimensional; and thirdly, the control's value depends on the population of the object's states at the measurement points; specifically, the multi-index identifying the phase space zone (hypercube) containing the current measurement values determines the control's value. Synthesized zonal controls solve the feedback control problem (1) – (6), as long as the selection of the control's values and feedback with the object is made at the given discrete times.

III. Gradient Formulas

The approach described in [16] can be used to solve the formulated zonal control problem (1) – (6) numerically. To this end, the application of efficient first-order numerical optimization methods, such as the conjugate gradient projection method [17 – 19], is proposed because the set of admissible control values U has a simple structure. According to the conjugate gradient project method, a minimizing sequence of approximations $\{v^k\}$ is built as follows:

$$v^{k+1} = \mathrm{Proj}_U(v^k + \alpha_k \times p^k),\ \alpha_k > 0,\ k = 0,1,2, \ldots,$$

$$p^k = -\nabla F(v^0),\ p^{k+1} = -\nabla F(v^k) + \beta_k \times p^k,$$

$$\beta_k = \|\nabla F(v^{k+1})\|_{\mathbb{R}^{r^n}} / \|\nabla F(v^k)\|_{\mathbb{R}^{r^n}},$$

where $v^0 \in \mathbb{R}^{r^n}$ is some initial guess; k the iteration number of the iterative procedure; $\nabla F(v^k)$ the objective functional's gradient; α_k the step taken in the descent direction p^k; $\mathrm{Proj}_U(\cdot)$ the projection operator onto U. The posed problem can also be solved using methods of sequential unconstrained minimization (e.g., methods of external and internal penalty functions based on unconstrained quasi-Newtonian methods [20, 21]), especially in cases when the dimension of the problem is relatively high or if the set U has a complex shape, and the projection operator onto U does not have constructive characteristics. Having exact formulas for the objective function's gradient with respect to the optimizable parameters is essential for constructing iterative procedures based on the above-mentioned optimization techniques. Therefore, the research continues with deriving formulas for the gradient using the method for computing the objective functional's increment generated by incrementing the optimizable parameters' values. The following observation is significant for further calculations. The components of the set Γ are independent of each other, and hence, the following relation holds for the gradient:

$$\nabla F(v) = \int_{\mathrm{T}} \nabla \Phi(v, \mathcal{T}_0)\, \rho_{\mathrm{T}}(\mathcal{T}_0)\, d\mathcal{T}_0.$$

Consequently, formulae for the gradient of $\Phi(v, \mathcal{T}_0)$ with respect to a single term $\mathcal{T}_0$ are derived to obtain formulas for $\nabla F(v)$. This is done by calculating the increment of the functional $\Phi(v, \mathcal{T}_0)$ generated by incrementing each of the components of the vector

$$v = v_{(i_1, i_2, \ldots, i_n)} = (v_1, v_2, \ldots, v_{r^n}),$$

$$i_s \in \{1,2, \ldots, r\},\ s = 1,2, \ldots, n.$$

The following theorem is true.

Theorem 1. *In the feedback control problem (1) – (5), for an arbitrary admissible control $v \in U$, in the space of piecewise constant controls (6), the components of the objective functional's gradient are determined by the formula:*

$$\frac{\partial F(v)}{\partial v_\ell} = -\int_{\mathrm{T}} \int_{\Gamma_{i_1, i_2, \ldots, i_n}(v_\ell)} \int_0^1 b\, \psi(x, t) dx\, dt\, \rho_T(\mathcal{T}_0)\, d\mathcal{T}_0,$$

$$\Gamma_{i_1, i_2, \ldots, i_n}(v_\ell) = \cup_{\overline{\mathcal{T}}(t_j) \in Z_{i_1, i_2, \ldots, i_n}} [t_j, t_{j+1}), \quad (7)$$

where $\Gamma_{i_1, i_2, \ldots, i_n}(\cdot)$ is the union of separate time intervals because, generally speaking, the object's state $\overline{\mathcal{T}}(t)$ can be in the same zone at different periods $[t_j, t_{j+1})$, $j \in \{0,1,2, \ldots, m-1\}$, i.e., the system's trajectory may enter and exit the same zone more than once; $\psi(x, t; v, \mathcal{T}_0)$ denotes the solution of the adjoint problem

$$b\, \psi(x, t) - \frac{\partial \psi(x,t)}{\partial t} - a \frac{\partial^2 \psi(x,t)}{\partial x^2} = 0,\ x \in [0,1],\ t \in [0, \tau) \quad (8)$$

satisfying initial and boundary conditions

$$\psi(x, \tau) = -2w(x)[\mathcal{T}(x, \tau) - \mathcal{T}^{\star}(x)],\ x \in [0,1], \quad (9)$$

$$\frac{\partial \psi(0,t)}{\partial x} = \frac{\partial \psi(1,t)}{\partial x} = 0,\ t \in [0, \tau], \quad (10)$$

respectively, for the particular initial condition $\mathcal{T}_0$ and current zonal control v.

The existence and uniqueness of generalized solutions to the adjoint problem (8), (9), (10), as well as to the direct problem (1), (2), (3) under the adopted assumptions on the functions involved in the problems, with the dimension of the spatial variable less than or equal to 3, are shown in [2, 15]. The following iterative procedure for optimizing piecewise constant controls (6) can be proposed based on first-order numerical optimization techniques using the formulae for the objective functional's gradient (7).

Step 1: Using a numerical technique, the direct boundary-value problem for (1) is solved for the current vector $v \in U$ and all the initial conditions $\mathcal{T}_0 \in \mathrm{T}$. This allows for determining the phase trajectory $\mathcal{T}(x, t; v, \mathcal{T}_0)$.

Step 2: The adjoint boundary-value problem (8) – (10) is solved numerically, and the trajectory $\psi(x,t;v,\mathcal{T}_0)$ determined.

Step 3: The objective functional's gradient components are evaluated by formulas (7) using a numerical integration technique.

Step 4: First-order finite-dimensional optimization numerical algorithms, such as the (conjugate) gradient projection method, are applied to calculate even better approximations of the control vector.

Step 5: Steps 1 – 4 are repeated if an optimality condition is unmet. Otherwise, the iterative process ends, for example, when the following optimality condition is satisfied for a relatively small positive quantity ε:

$$|F(v^{k+1}) - F(v^k)|/(1 + |F(v^{k+1})|) < \varepsilon.$$

The requirements for organizing feedback in object control, set out in the following two remarks, are crucial for the practical application of the proposed approach to zonal control synthesis.

Remark 1. It is important to choose the observation times t_{j}, $\mathrm{j} = 0,1,2,\dots,m$, in such a way that the object's state would be observed in a zone at least once if the phase trajectory ever passes through that zone. Zonal control parameters cannot be assigned to zones through which the phase trajectory did not pass under all possible initial conditions and zones where no observations were ever made.

Remark 2. The selection of the number of zones (5) and their configuration significantly impacts the control system's quality. A refinement of zones can only reduce the value of the objective functional. By increasing the number of zones, control actions can change their values more frequently in time, which leads, on the one hand, to a deterioration of the robustness of the control system and, on the other hand, to rapid wear of the actuating mechanisms. However, reducing the number of zones or increasing their size, on the other hand, will deteriorate the object's controllability, and with fewer zones, it may become completely unpredictable. As a result, the objective functional's value increases, so the quality of control deteriorates. It is recommended to start by arbitrarily selecting an initial value of r and assigning some zones (5). All neighboring zones' optimal zonal values can be evaluated once the feedback control problem has been solved. Two adjacent zones with a sufficiently small difference in optimizable parameters can be combined into one, which reduces the number r. However, the feedback control problem should be revisited if the optimizable parameters in any two adjacent zones differ significantly, and each of them should be divided, for instance, into two so that the number r is increased. The number of zones should be increased until the objective functional's optimal value no longer changes (decreases) significantly.

The proposed approach to control feedback is characterized by the high dimensionality of the optimizable control vector because it represents an exponential function with respect to the number of thermal sensors and a power function with respect to the number r of temperature intervals within the range $[\mathcal{T}_{\min},\mathcal{T}_{\max}]$. Computing optimal solutions of high-dimensional objective functions is recognized as one of the fundamental issues with numerical optimization techniques (of any order) due to the high computing cost and complicated nature of optimizing high-dimensional objective functions. Calculating optimal values of zonal feedback parameters, as is easy to see, leads to a complex computational problem, the numerical solution of which requires large computing resources. Considering this, the following recommendation suggests a way to speed up the process of finding a solution to improve the efficiency of the solution process. The inherent concurrency in the objective functional of the posed feedback control problem can be exploited to speed up its computation. Its computation can be efficiently parallelized by assigning each thread (or process) several elements of the set T and computing the innermost definite integral in (4) with high accuracy since the elements of the set T are independent of each other. The same holds for the solution to both the direct and adjoint boundary-value problems and the evaluation of the objective functional's gradient by (7) [22, 23].

IV. Results of Numerical Experiments

As an example of solving the control synthesis problem for a parabolic system, let us consider the following boundary-value problem:

$$\frac{\partial\mathcal{T}(x,t)}{\partial t} = \frac{\partial^2\mathcal{T}(x,t)}{\partial x^2} + [u(t) - \mathcal{T}(x,t)],\ x \in (0,1),\ t \in (0,10],$$

subject to the boundary conditions

$$\frac{\partial\mathcal{T}(0,t)}{\partial x} = \frac{\partial\mathcal{T}(1,t)}{\partial x} = 0,\ t \in [0,10],$$

and the initial conditions:

$$\mathcal{T}(x,0) = \mathcal{T}_0 = \text{const},\ x \in [0,1],$$

where the temperature's initial distribution $\mathcal{T}_0$ belongs to the discrete set $\mathrm{T} = \{0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0\}$.Two thermal sensors are installed at the points $x_1 = 0.3$ and $x_2 = 0.7$, which are used to observe the heating process and feed data into the control system in the form of a vector $\overline{\mathcal{T}}(t) = (\overline{\mathcal{T}}_1(t);\overline{\mathcal{T}}_2(t)) = (\mathcal{T}(0.3,t);\mathcal{T}(0.7,t))$. Suppose the object's state can be measured at the points x_1 and x_2 continuously in time. The heat conduction process is controlled by synthesizing a regulator that maintains the temperature $\mathcal{T}(x,\tau)$ at the desired level $\mathcal{T}^\star(x) = 9.0$ by adjusting the heat source's temperature $u(t)$ based on the temperature measurements sampled at the points x_1 and x_2. Assume that the admissible control values belong to the set $U = \{u(t)\colon\ 0 \le u(t) \le 10,\ t > 0\}$. The objective functional to be minimized in the feedback control problem is as follows:

$$F(u) = \int_{\mathrm{T}} \Phi(u,\mathcal{T}_0)\cdot\rho_{\mathrm{T}}(\mathcal{T}_0)\,d\mathcal{T}_0,$$

$$\Phi(u,\mathcal{T}_0) = \int_0^1 w(x)[\mathcal{T}(x,\tau;u,\mathcal{T}_0) - \mathcal{T}^\star(x)]^2 dx,$$

where the weight function $w(x) = e^{-(x-0.5)^2}$ gives points close to the center of the rod's length more weight in the resulting integral value than points farther from the center. Conducted numerical experiments show that the rod's phase state values satisfy the following inequality

$$0 \le \mathcal{T}(x,\tau;u,\mathcal{T}_0) \le 10,\ x \in [0,1],\ t \in [0,10],$$

for all possible combinations of initial and given boundary conditions and the control's admissible values. The range of possible temperature values, $[0,10]$, is divided into 12 intervals

$$[\mathcal{T}_{\min}, \mathcal{T}_{\max}] = \cup_{k=1}^{12}(\xi_{k-1}, \xi_k), \xi_0 = -\infty, \xi_{12} = +\infty,$$

by the partition points $\xi_k = k-1$, $k = 1,2,\ldots,11$. The following rectangles (zones) in the phase plane $\overline{\mathcal{T}}(\cdot) \in \mathbb{R}^2$ are introduced:

$$\mathrm{Z}_{i_1,i_2} = \{(\overline{\mathcal{T}}_1, \overline{\mathcal{T}}_2): \xi_{i_s-1} \le \mathcal{T}(x_s,t) \le \xi_{i_s}\},$$

$$s = 1,2,\ i_s \in \{1,2,\ldots,12\},$$

with the total number of zones equal to 144. Let the two-dimensional multi-index $\mathrm{I} = (i_1, i_2)$ determine the number of the corresponding zone. Each zone is associated with its constant control value, i.e.,

$$u(t) = v_{i_1,i_2} = v_{\mathrm{I}} = \mathrm{const},\ \overline{\mathcal{T}}(t) \in \mathrm{Z}_{i_1,i_2} = \mathrm{Z}_{\mathrm{I}},$$

To start the minimization process, 144 random values were taken from the interval $[0,10]$ as an initial guess for the optimizable vector. The objective functional's value was reduced to approximately 10^{-8} after running the minimization procedure. Note that the minimum possible value the functional can assume is 0.

V. Conclusion

This paper presented an approach to the numerical solution of the feedback control problem applied to a distributed parameters object, represented by a one-dimensional parabolic equation given inaccurate information on the values of the object's initial conditions. The solution scheme applies the gradient-based numerical methods to the reduced finite-dimensional optimization problem constructed from the initial problem using the principle of zonality. To this end, formulas for the objective functional's gradient have been derived. It is known that piecewise-constant synthesizing functions can be implemented technically rather easily and with a high enough degree of precision. Such an approach to synthesizing control action parameters increases the robustness of the control system and extends the life cycle of all equipment involved in the control circuit, including actuators. A wide range of automatic and automated control systems can benefit from the proposed method for solving control synthesis problems in the presence of imprecise knowledge about the object's state. Numerous controllable mechanical systems and technological processes whose states are described by nonlinear differential equations systems can be the application objectives of such systems. Heat conduction processes in two and three dimensions can easily be studied using the proposed approach.

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