A Systematic Approach to Thermal System Modeling with Examples Using First Principles

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***Abstract*—Accurate thermal system modeling is critical for advancing control simulations and optimizing performance across a wide range of applications. Despite its importance, many existing studies rely on empirical models or provide limited explanations, hindering a deeper understanding of the modeling process. This paper highlights the gap in the field by analyzing existing works that used fuzzy-PID control in thermal system modeling, pointing out the lack of first-principles-based approaches. The paper presents an explanation of fundamental concepts such as thermal resistance and capacitance, illustrated with practical heat transfer examples. It then proposes that the plant in thermal systems should be viewed as two separate blocks: the actuator, which consists of electrical equipment and components, and the environment, represented as the compartment. The role of these components in control system design is discussed, emphasizing the need for distinct models of the actuator and environment for more accurate simulations and improved thermal regulation. This work underscores the need for a more systematic approach to thermal modeling, providing a clearer path for future advancements in both theory and practical applications.**

***Keywords*—temperature control, heat exchange process, thermal system, thermal models, fuzzy-PID**

I. Introduction

Thermal systems are an integral part of various engineering applications, including HVAC design, electronic equipment thermal management, and energy-efficient building systems. Understanding and accurately modeling thermal systems is fundamental to various engineering disciplines, yet it often remains an underexplored area in academic and industrial contexts. Thermal modeling involves analyzing the generation, transfer, and dissipation of heat in various systems, which is essential for optimizing performance and ensuring system reliability. Despite its critical importance, many existing studies rely on simplified assumptions or empirical methods [1-12], often overlooking the systematic derivation of models from first principles. This paper aims to address this gap by presenting a structured framework for developing thermal models based on fundamental physical laws, such as thermal resistance and capacitance. It emphasizes the importance of grounding these models in first principles for more accurate and reliable system analysis.

This paper provides a systematic exploration of thermal system modeling, focusing primarily on deriving models from first principles while briefly touching on other approaches, such as using existing models and system identification techniques as examples of how to approach well-behaved systems. It explains the foundational concepts of thermal resistance and capacitance, demonstrates their application through examples such as a thermometer in a liquid, room heat transfer, and modeling of electrical components, and illustrates how to combine these models to represent complex equipment. Additionally, it evaluates existing works in the field to highlight practical applications and insights, equipping readers with the knowledge to apply these methods effectively and address challenges in thermal system design and analysis.

II. Use of Thermal Models

For a well-behaved system, there are three main ways of deriving a mathematical model (fig. 1). These are using an already existing model, deriving a model by understanding the first principles of any given system, and system analyses. The easiest way is to use an already existing model. However, this method is very limited and may cause problems when designing a controller or making even slight adjustments to the system. Thus, understanding the underlying mechanics of the simulated system is very important.

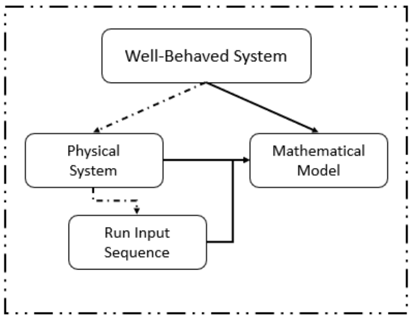

Fig. 1. Ways of deriving models for well-behaved systems

In the case that the physical parameters of the system are unknown or difficult to measure, the system analysis method is used. This is done by subjecting the system to controlled inputs and measuring its outputs to identify the underlying dynamics. For instance, observing the temperature response of a room to a heater's operation can reveal the time constant and steady-state gain, which can be used to construct a simplified thermal model. Studies made by [1-12] show the use of fuzzy-PID controllers in temperature measurements of various applications. Their studies show that fuzzy-PID can significantly improve the performance and robustness of the system by tuning the proportional integral derivative (PID) controller's parameters in real time. A generalized block diagram of the fuzzy-PID controller system is shown below in (fig. 2).

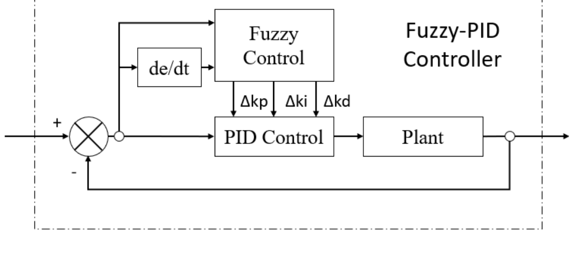

Fig. 2. General block diagram for fuzzy-PID controller

In all of these studies, it can be seen that the main goal is to tune the PID controller first, and then use the fuzzy logic controller (FLC) to adjust its gains based on the error to tune the PID gains automatically while the control process is ongoing. For a better understanding of the working principle here, let's first take a brief look at how the output signal of a PID controller is generated [1-13]:

$$U_t = K_p \cdot e(t) + K_i \cdot \int_0^t e(\tau)d\tau + K_d \cdot \frac{de(t)}{dt} \qquad (1)$$

Here, three different branches contribute to the output signal of a PID controller. The integral path is responsible for remembering the past, while the derivative path looks ahead into the future, and the proportional acts for the current situation [13]. The role of the FLC is to give initially tuned gains K_0 an additional adjustment ΔK based on the error for an even better response:

$$\begin{cases} K_p = K_{p0} + \Delta K_p \\ K_i = K_{i0} + \Delta K_i \\ K_d = K_{d0} + \Delta K_d \end{cases} \qquad (2)$$

The reviewed works illustrate the prevalent reliance on empirical methodologies for modeling and control across diverse systems, highlighting the gap in approaches grounded in first principles. These examples underscore the limitations of data-driven methods in capturing the fundamental physical behavior of thermal systems.

For instance, in [1], a turbine flowmeter determines cooling water flow through pulse frequency measurements, relying on empirical relationships rather than physical laws. Similarly, in [2], the evaporator of an organic Rankine cycle system is modeled as a first-order inertial system, with the transfer function identified experimentally to optimize system efficiency, foregoing derivation from thermal or fluid dynamics principles.

Empirical methods also dominate in control system optimization. In [3], an existing mathematical model of an indoor system is enhanced with a fuzzy-PID controller, focusing on improved performance without revisiting the system's foundational physical characteristics. In [4], the ground source heat pump terminal fan subsystem is modeled using simplified representations, such as proportional or first-order inertial elements, derived through component dynamics and experimental data rather than first principles. In [5], fractional-order differential equations describe a heat exchanger process, with parameters estimated using the Grunwald-Letnikov method, avoiding derivation from conservation laws.

The trend continues in [6], where the Chemostat system is modeled with a linearized, unstructured cell growth model for simplified analysis and control, bypassing fundamental derivations. Likewise, in [7], a thermal object's transfer function is empirically derived using experimental data, focusing on response accuracy over theoretical derivation. In [8], a state-space model is developed for a heating system, connecting outputs and inputs across phases using empirical relationships rather than foundational thermodynamic principles.

Industrial applications further demonstrate this empirical focus. For example, [9] models a semiconductor chiller's transfer function using operational data, while [10] designs and calibrates a temperature control system for a copper block with an optical crystal, relying on experiments to establish stability and performance. Similarly, in [11], control strategies for a three-level active neutral point clamped inverter are compared using simulations and experiments, emphasizing empirical methods over fundamental derivations.

In some cases, a hybrid approach is necessary, combining first principles with experimental data. This is particularly useful when simplifying assumptions is insufficient to capture system behavior accurately. By calibrating first-principles models with experimental data, we achieve a balance between accuracy and practicality. Like in [12], which models an underwater spherical robot's dynamics using first principles from rigid body and fluid mechanics. However, even here, significant simplifications are made, such as decoupling motion and assuming symmetry to make the model practically usable.

These studies reveal a consistent reliance on empirical techniques for modeling and controlling thermal and dynamic systems. While these methods are often effective for specific applications, they fall short of providing the robustness and generalizability offered by models derived from first principles. The final and often overlooked method is to derive models using first principles. For thermal systems, this involves leveraging thermal resistivity, which quantifies resistance to heat flow, and thermal capacitance, representing the ability to store heat. By applying Fourier's law and energy conservation principles, it becomes possible to derive governing differential equations that provide a universal and reliable framework for understanding and modeling various thermal systems.

III. Deriving a Thermal Model

The primary advantage of linear systems lies in the vast array of analytical tools and computational methods developed for their modeling and simulation. These resources streamline the design and analysis processes, enabling precise predictions and efficient optimization of system behavior. In thermal systems, mathematical models act as the plant block within a closed-loop control system (fig. 3). This role is pivotal in simulating system dynamics and providing a foundation for designing controllers that effectively regulate temperature under varying operational conditions.

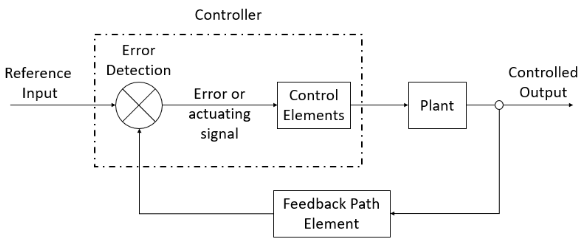

Fig. 3. General block diagram of closed-loop control system

The plant itself comprises two essential components: the actuator and the process (fig. 4). The actuator is responsible for manipulating the controlled variable and facilitating precise adjustments within the process to achieve the desired system behavior.

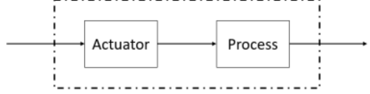

Fig. 4. Plant block components

It is crucial to recognize that both the actuator and the process require distinct mathematical models for accurate simulations. As a result, these components are typically modeled separately in most applications. Actuators, which are usually man-made devices composed of electronic components, are designed to manipulate the controlled variable. Meanwhile, the process typically represents a system, such as a room-like environment. However, not all systems can be effectively represented using conventional mathematical models. This limitation arises when the system is either overly complex or exhibits significant nonlinearity, even under constrained operating conditions. For such inherently nonlinear systems, advanced modeling techniques are necessary, often relying on approximations or heuristic methods to achieve practical and usable results.

A. Building Blocks

Many systems can be effectively represented by first-order differential equations. In thermal systems, the mathematical model is typically derived using just two fundamental building blocks: thermal resistance and thermal capacitance [14,15]. The value of the thermal resistance depends on whether the heat transfer is taking place through convection or conduction. For unidirectional conduction through a solid material, the equation will be:

$$R_{Conduct} = \frac{\Delta x}{K \cdot A} \tag{3}$$

Here, A is the cross-sectional area of the material through which the heat is being conducted, K is the thermal conductivity of that material, and Δx is the thickness of the material between two given points. For convection which is the heat transfer between liquids and gasses, the thermal resistance is calculated by:

$$R_{Convect} = \frac{1}{h \cdot A} \tag{4}$$

Thermal capacitance, on the other hand, is the measure of stored internal energy in a system. It is represented as:

$$C = m \cdot c \tag{5}$$

Where m is mass, c is specific heat capacity.

B. Heat Transfer Process

Let's assume we have a container filled with liquid with the temperature T_L (fig. 5). We insert a thermometer inside this liquid to measure heat temperature, which results in the heat transfer from liquid to thermometer [14,16].

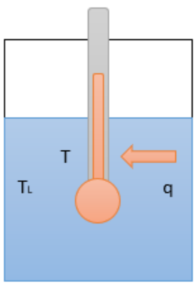

Fig. 5. Thermometer inside a liquid tank

The rate of heat transfer between the liquid and thermometer is proportional to the temperature difference between them and inversely proportional to the thermal resistance:

$$Q_{rate} = \frac{T_L - T_T}{R} \tag{6}$$

Where Q is the heat transfer rate, T_L is the temperature of the liquid, T_T is the temperature of the thermometer, and R is the thermal resistance between the liquid and thermometer. The face of the thermometer getting warmer also needs to be addressed, and since absorbed heat changed the temperature according to the thermal capacitance, the relationship for the thermometer will be as follows:

$$Q_{rate} = C_T \cdot \frac{dT_T}{dt} \tag{7}$$

Where C_T is the thermal capacitance of the thermometer, which represents how much heat is required to changing the thermometer by one degree. Now, we can equate these two equations together and derive the final differential equation:

$$\frac{dT_T}{dt} = \frac{T_L - T_T}{R \cdot C_T} \tag{8}$$

This equation shows how the temperature of the thermometer increases over time due to the heat transfer from the liquid, with the rate of temperature change being proportional to the temperature difference and the thermal capacitance of the thermometer. Finally, we need to calculate the thermal resistance here. However, for this, we need to account for two resistances:

$$R = R_T + R_L \tag{9}$$

Here, R_T is the thermal resistance of the thermometer, and its mode of heat transfer is conduction, while R_L represents the thermal resistance of the water, which is liquid and its mode of heat transfer is convection. Thus, the equation can be written as:

$$R = \frac{L_T}{K_T \cdot A_T} + \frac{1}{h \cdot A_L} \tag{10}$$

Here, L_T is the length or diameter, K_T is the thermal conductivity, and A_T is the cross-sectional area of the thermometer, while A_L is the surface area of the thermometer in contact with the liquid of the liquid, and h is the convective heat transfer coefficient between the thermometer and the liquid.

C. Modeling a Thermal Process

Let's suppose we have a heater located inside the room that gives the heat flow of Q_1 (fig. 6). Because of this, there is a room temperature offset of T, and the thermal capacitance of this room is C. Beyond the room, on the other side through the wall there is a heat transfer Q_2, and the surrounding temperature is T_0. The representation of this room is given below [14,16].

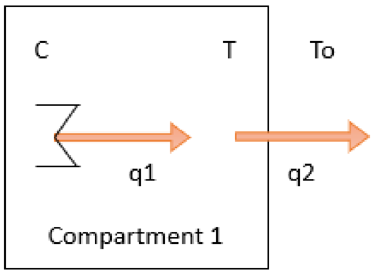

Fig. 6. Compartment with a single heat source inside

Here, the room is being heated with an electric heater and losing heat through the insulation of the room. If we assume that, there is also no heat storage by the walls. The equation of compartments temperature change can be written by its thermal capacity formula:

$$C \cdot \frac{dT}{dt} = Q_1 - Q_2 \tag{11}$$

From here, the heat loss Q_2 can be defined by the following formula:

$$Q_2 = \frac{T - T_0}{R} \tag{12}$$

Here R is the thermal resistance of the walls and other materials separating the room from the outside. From here, by dividing each side by the thermal capacitance, we can get the formula for the change in temperature:

$$\frac{dT}{dt} = \frac{Q_1}{C} - \frac{T - T_0}{R \cdot C} \tag{13}$$

Once the differential equation is obtained, the focus here can be shifted towards the thermal resistance:

$$R = R_1 + R_2 + \cdots + R_n \tag{14}$$

Where each R represents the thermal resistance of each unique layer of the walls, and for each layer of the wall, thermal resistance can be calculated using the conduction equation:

$$R_{Layer} = \frac{\Delta x}{K \cdot A} \tag{15}$$

D. Modeling of Actuators Used in Thermal Systems

Actuators used in thermal systems consist of multiple components that generate heat during operation. The thermal behavior of these components can be modeled using fundamental principles of heat transfer. The heat generated by a single component can be expressed as:

$$\frac{dT_T}{dt} = \frac{Q_i}{C} - Q_{Loss} \tag{16}$$

Here, Q_i is the heat generated by the component, C represents the thermal capacitance, and Q_{loss} accounts for the heat lost to the environment. The heat loss to the surrounding environment can be modeled using a thermal resistance-based approach, expressed as:

$$Q_{Loss} = \frac{T - T_\infty}{R_{Thermal}} \tag{17}$$

In this equation, T is the component's temperature, T_∞ represents the ambient temperature, and $R_{Thermal}$ is the thermal resistance of the medium through which heat is transferred. The complete thermal model of the actuator, which includes all components and their interaction with the environment, is given by:

$$\frac{dT_{actuator}}{dt} = \frac{Q_{gen}}{C_{actuator}} - \frac{T_{actuator} - T_\infty}{R_{system}} \tag{18}$$

Here, Q_{gen} is the total heat generated by all components, $C_{actuator}$ is the thermal capacitance of the actuator as a whole, and the R system represents the combined thermal resistance between the actuator and the environment. Finally, the total heat generation Q_{gen} within the actuator is the sum of the heat generated by its individual components, expressed as:

$$Q_{Gen} = \sum_{i=1}^{N} Q_i \tag{19}$$

This formulation allows the thermal model of the actuator to account for the contributions of each component, enabling an accurate representation of its thermal dynamics.

E. Modeling of Actuators Components

In a thermal system, the total heat generation within an actuator comes from various components, each contributing differently to the overall thermal load. The most common sources of heat generation are electronic components, motors, and resistive elements. For resistors, the heat generation is primarily due to Joule heating, which can be expressed as:

$$Q_{Gen} = I^2 \cdot R = \frac{V^2}{R} \tag{20}$$

Where I is the current passing through the resistor, and R is its resistance. For semiconductors or transistors, the heat generated is typically due to power dissipation when the device operates in a non-ideal state. This can be expressed as:

$$Q_{Gen} = V_{DS} \cdot I_{DS} \tag{21}$$

Where V_{ds} is the drain-source voltage and I_{ds} is the drain-source current. Motors, especially electric motors, generate heat due to both resistive losses in the windings and frictional losses from moving parts. The heat generation from a motor can be modeled as:

$$Q_{Gen} = I^2 \cdot R + P_{\mathrm{friction}} \tag{22}$$

Where I is the current through the motor windings, R is the resistance of the windings, and $P_{friction}$ represents the power lost to frictional forces. In capacitors, heat generation mainly arises from dielectric losses, which depend on the operating frequency and the properties of the dielectric material. The heat generation from a capacitor can be approximated as:

$$Q_{Gen} = I_{rms}^2 \cdot R_{loss} \tag{23}$$

Where I_{rms} is the root mean square current, and R_{loss} is the equivalent series resistance (ESR) of the capacitor. For inductors, the heat generation is due to core losses and resistive losses in the windings:

$$Q_{Gen} = I^2 \cdot R_{Wind} + P_{core\ losses} \tag{24}$$

Where I is the current flowing through the inductor, R_{wind} is the resistance of the windings, and $P_{corelosses}$ represents losses due to magnetic hysteresis and eddy currents in the core material. Additionally, power supplies or voltage regulators generate heat due to conversion inefficiencies:

$$Q_{Gen} = P_{in} - P_{out} \tag{25}$$

Where, P_{in} is the input power and P_{out} is the output power, representing the inefficiencies of the power conversion process.

F. Example of a Heater's Model

The Lasko 754200 Ceramic Heater is a good example for thermal system modeling due to its accessibility, as it can be both tested through simulations and easily purchased by everyday users for validation purposes [17]. The thermal dynamics of the system can be summarized as:

$$C_{th} \cdot \frac{dT_h}{dt} = Q_{gen} - Q_{loss} \tag{26}$$

Here, C_{th} is the thermal capacitance, T_h is the heater surface temperature, Q_{gen} represents the heat generated by the heater, and Q_{loss} accounts for the heat lost to the environment. The heat loss depends on the ambient temperature and thermal resistance, while the heat generation comes from a ceramic heating element that incorporates positive temperature coefficient (PTC) ceramic resistors. From here, by substituting everything discussed earlier, the equation will be:

$$C_{th} \cdot \frac{dT_T}{dt} + \frac{T_h}{R_{th}} = \frac{V^2}{R_e} - \frac{T_{ambient}}{R_{th}} \tag{27}$$

Where, R_{th} is the thermal resistance, which governs how heat is transferred from the heater to the surrounding air. R_e is the electrical resistance of the heating element, determining how efficiently electrical energy is converted to heat. $T_{ambient}$ is the ambient temperature, affecting heat loss to the environment. From here, we can get the mathematical representation of the temperature response of the heater in the frequency domain:

$$T_h(s) \cdot \left(C_{th} \cdot s + \frac{1}{R_{th}}\right) = \frac{V^2}{R_e} - \frac{T_{ambient}}{R_{th}} \tag{28}$$

Finally, we can solve for trasnfer fucniton G(s) by getting the ratio of the output temperature response to the input heat generation term:

$$G(s) = \frac{T_h(s)}{\frac{V^2}{R_e}} \tag{29}$$

By substuting the $T_h(s)$ value, the equaiton will be:

$$G(s) = \frac{1}{C_{th} \cdot s + \frac{1}{R_{th}}} \cdot \left(1 + \frac{\frac{T_{ambient}}{R_{th}}}{\frac{V^2}{R_e}}\right) \tag{30}$$

Here, the second term is the influence of the ambient temperature, but it's not the primary focus for defining the system's dynamic response, which is why it is typically excluded when discussing just the thermal transfer function:

$$G(s) = \frac{1}{C_{th} \cdot s + \frac{1}{R_{th}}} \tag{31}$$

IV. Conclusion

In conclusion, while many studies have focused on empirical methods for modeling and controlling thermal systems, this research fills a crucial gap by introducing a systematic, first principles-based approach. By breaking down the plant into two components, the actuator, and the process, and modeling both from fundamental physical principles, this work provides a comprehensive framework for understanding and analyzing thermal systems. Additionally, a stepped guide for deriving the transfer function is presented, incorporating key concepts like thermal resistance, capacitance, heat transfer, and the thermal models of the actuator and process. This approach contributes to both theoretical insights and practical applications in the field.

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