Data Structures and Route Reduction Procedures in the Problem of Distribution and Routing of Flows in a Communication Network

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***Abstract*—In the problems of the distribution and routing of flows in communication networks, the input data is vehicle routes or data transmission channels. For the distribution of flows in this problems in optimization algorithms use special data structures – abstract data types, which describe the relationship between distributed flows and routes, as well as route reduction procedures, which can significantly reduce the number of such connections. The paper develops data structures and route reduction algorithms that allow solving the problem of distribution of flows in the case when the number of specified routes is very large, and the amount of computer RAM is limited. Estimates of the time complexity of the algorithm for reducing routes by nodes and arcs, as well as the algorithm for generating a reference data structure, have been obtained. The proposed algorithms were tested on networks with the number of nodes from 50 to 500 and the number of routes from 1225 to 124750, which showed their performance, good computational efficiency and they can be used in practical problems of distribution and routing of flows on large-dimensional networks.**

Keywords—multicommodity hierarchical networks, discrete flows, problems of combinatorial optimization, computer modeling

I. Introduction

In the problems of optimizing the distribution and routing of flows in communication networks, the input data is vehicle routes or data transmission channels [1]. In terms of graph theory, a route is a simple path given by the enumeration of nodes and arcs of a network. Circular routes in which the initial and final nodes coincide are allowed. A set of routes can be initially specified or generated according to certain rules in the process of solving a problem. Because routes have different lengths in terms of the number of nodes (or arcs) included in them, shorter routes can coincide with longer routes, both in nodes and in arcs. For the distribution of flows in such problems, special data structures are used – abstract data types (ADT) [2, 3], with the help of which the relationship between distributed flows and routes is described. Route reduction procedures can significantly reduce the number of such connections in the case when the number of initial or generated routes in combinatorial optimization problems is very large.

The article discusses data structures and procedures for route reduction, which are used in algorithms for solving the problem of optimization of distribution and routing of flows [1]. The developed algorithms for route reduction allow solving the problem of distributing flows when the number of specified routes is too many, and the amount of RAM of the computer is limited. Estimates of the complexity of algorithms for reducing routes by nodes and arcs, as well as the algorithm for forming a reference data structure, are obtained. The proposed algorithms were tested on networks with the number of nodes from 50 to 500 and the number of routes from 1225 to 124750, which showed their operability, good computational efficiency and they can be used in practical problems of distribution and routing of flows on large-dimensional networks.

II. Data Structures and Route Reduction Procedures

Let $G(N,P)$ – a multicommodity network with a set of undirected topological arcs P, $p = |P|$, a set of nodes N, $n = |N|$. A topological arc is a physical segment of a communication line. Network nodes correspond to the points of departure, receipt and transshipment (switching) of flows.

A matrix of flows is defined on the network $U = \| u_{ij} \|_{n\times n}$, $i, j = \overline{1,n}$. Flows u_{ij} from sources i into the drains j, $i, j = \overline{1,n}$ must be transported in vehicles or transmitted through communication channels. A set of vehicle routes or communication channels is also specified $\{m_k\}$, $k = \overline{1,l}$, each of which consists of a sequence of nodes and topological arcs of the network G, connecting the start and end nodes of a route or communication channel. In a particular optimization problem, the set of routes may not be specified, but searched. For data networks, routes can be represented by simple communication channels connecting adjacent nodes or switched communication channels connecting any sequence of nodes (dedicated channels). In the individual case, all specified routes can coincide with the topological arcs of the network. Plural $\{m_k\}$ can contain multiple routes, nodes connecting any pair. An additional route multi-network is defined $G_M(N,P_M)$, where N – a set of network nodes, P_M — the set of its oriented route arcs. Between any nodes i and j network G_M there is a route arc if they are connected by at least one vehicle route or communication channel with $\{m_k\}$.

As you know, the computational efficiency of algorithms largely depends on the successful representation of the data structures used in the process of solving the problem. When solving the problem of distribution and routing of flows, it is necessary to have such a data structure that, on the one hand, would allow for each flow u_{ij} quickly find the set of routes to which this flow can be allocated, as well as define the set of flows that can be implemented on this route. On the other

hand, the data structure should ensure the storage of the results of solving the problem – a flow distribution scheme in which for each u_{ij} defined path from the original i-th node to the end j-th node with directions and transit nodes. Another important requirement for the organization of the data structure is the minimum of information stored in the process of solving the problem.

Let $\upsilon_k = \{\xi_1, \xi_2, ..., \xi_{\eta_k}\}$ – an ordered set of nodes with N on the route m_k, $|\upsilon_k| = \eta_k$, $q_k = \{(\xi_1, \xi_2), (\xi_2, \xi_3), ..., (\xi_{\eta_k - 1}, \xi_{\eta_k})\}$, $|q_k| = \eta_k - 1$ – ordered set of arcs with P, that make up the route m_k.

Since in most cases the forward and return routes tend to be the same, in order to save computer RAM, it seems reasonable to keep only direct routes for coincident routes, bearing in mind that the transport of the flow along such a route is allowed in both directions. If the forward and return routes do not coincide, then each of them must be presented separately.

To distinguish the routes, we will set the sign of the direction of movement along the route. Let $\{m_k\}$, $k = \overline{1,l}$ – the set of routes, taking into account the comments made, and MV_k – indication of the direction of travel along the route, where

$$MV_k = \begin{cases} 0, \text{ if movement is allowed in both directions,} \\ 1, \text{ if movement is allowed in one direction.} \end{cases}$$

Let's also introduce additional features of the routes: $MT_k = \lambda$, where λ – an integer that defines the type of route;

$$MZ_k = \begin{cases} 0, \text{ if movement along the route is allowed,} \\ 1, \text{ if movement is forbidden.} \end{cases}$$

Additional features will be used in the algorithms for route reduction and the formation of a reference data structure.

Let's define the reference structure H_S, consisting of a matrix of pointers $\Lambda = \| \Delta_{ij} \|_{n \times n}$ and elements E, linked in unidirectional linear lists.

Index Δ_{ij} corresponds to the flow u_{ij} and points to the first element E, which is called the head of the list. Every element E may have a link to the next item. The last item in the list has a null reference ($NULL$ - link) and is called caudal. Every element E consists of a field FWD, that keeps the reference on the next element or value $NULL$; field AM route, indicating the address to which this flow can be distributed and feature fields DM. The feature field consists of five subfields, which take on the following values:

$$DM_1 = \begin{cases} 1, \text{ if the flow } u_{ij}, i < j \text{ is routed on } AM, \\ 0 \text{ otherwise;} \end{cases}$$

$$DM_2 = \begin{cases} 1, \text{ if the flow } u_{ij}, i > j \text{ is routed on } AM, \\ 0 \text{ otherwise;} \end{cases}$$

$$DM_3 = \begin{cases} 1, \text{ if for } u_{ij} \text{ is fulfilled } (i < j) \wedge (i \prec j), \\ 0, \text{ if for } u_{ij} \text{ is fulfilled } (i < j) \wedge (i \succ j); \end{cases}$$

$$DM_4 = \begin{cases} 1, \text{ if for } u_{ij} \text{ is fulfilled } (i > j) \wedge (i \prec j), \\ 0, \text{ if for } u_{ij} \text{ is fulfilled } (i > j) \wedge (i \succ j); \end{cases}$$

$$DM_5 = \begin{cases} 0, \text{ if the end of the chain of elements } E, \\ \text{defining the routes on which the flow } u_{ij} \\ \text{can be distributed without congestion is reached,} \\ 1 \text{ otherwise.} \end{cases}$$

Signs $i \prec j$, $i \succ j$ respectively imply that the node i precedes the node j, does it follow the node j when driving along the route AM in the order of passing the nodes with υ_{AM}.

Flow u_{ij} let's call it without transit if the following conditions are met for it: $\exists\, m_k \in \{m_k\}$, $k = \overline{1,l}$ for which it is true $(i, j \in \upsilon_k \wedge MV_k = 0) \vee (i, j \in \upsilon_k \wedge MV_k \neq 0 \wedge i \prec j)$. Otherwise, the flow will be called transit. At the initial formation of the structure H_S number of elements E on the list for no transit flow u_{ij} is determined by the number of routes to which the flow can be distributed, and if the flow is symmetrical u_{ji} also without transit, then the corresponding pointers Δ_{ij} and Δ_{ji} get a link to a single list. For transit flows that are to be transshipped one or more times from one route to another, the corresponding pointers from Λ zero links are obtained first.

Building chains of elements E for transit flows u_{ij} is carried out in the process of solving the problem and consists in the following: the routes to which the flow can be distributed are determined; for the found routes, elements are created E, which are linked to the list in accordance with the order of following the routes from i before j. In the information H_S for each transit the route node is written AM in which the flow is congested. Last item E in the chain for the transit flow u_{ij} in the field AY will contain a node j. In the case when, when solving the problem, branching (splitting) of flows into flow, only one list is remembered, which determines the path that was chosen to distribute this flow during the optimization process, intermediate lists are not remembered. If, in the process of solving the problem, the transit flow is transferred to the rank of transit, then the list selected for it is associated with the primary one. To differentiate the lists in this case, the subfield DM_5. For transit flows, each element E contains an additional field AY, to which the structure of elements is allowed E additional fields can be introduced to store a portion of the distributed flow.

Fig. 1 shows an example of a structure H_S which explains the concepts introduced. From the rules of initial formation H_S it is clear that the number of elements E in the lists H_S

depends on the number of routes and is defined as follows: $E_{HS} = \sum_{k=1}^{l} \eta_k (\eta_k - 1)/2.$

Magnitude E_{HS} for real-world networks can be very significant. So, for example, for a network containing 300 nodes when generated for all symmetrical $n(n-1)/2$ flows of the shortest paths of their distribution (individual routes) 44850 routes will be generated. With an average route length η_{cp} = 15 into the structure H_S will be included $44850\eta_{cp}(\eta_{cp}-1)/2 = 44850 \times 105 = 4709250$ elements that would require about 45 MB of RAM to accommodate (given that to accommodate one item E 10 bytes required).

In this regard, let's consider the reduction procedures that allow you to reduce the dimension of the structure H_S. Let the route m_α reduced to a route m_β by nodes, if the condition is met: $\upsilon_\alpha \cap \upsilon_\beta = \upsilon_\alpha$, and is reduced along topological arcs if $q_\alpha \cap q_\beta = q_\alpha$.

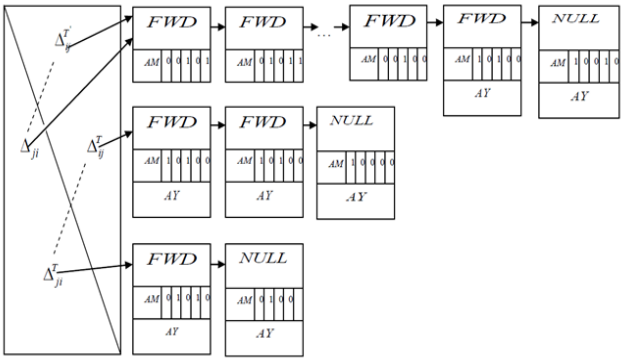


Fig. 1. Where: $\Delta_{ij}^{T'}$ – without transit flow, transferred to the rank of transit; Δ_{ji} – without transit flow; Δ_{ij}^{T}, Δ_{ji}^{T} – symmetrical transit flows

Obviously, route reduction reduces the number of connections in the network G_M and the number of elements in the structure H_S by the amount: $\sum_{\alpha \in \{k^*\}} \eta_\alpha (\eta_\alpha - 1)/2$, where $\{k^*\}$ – the set of numbers of reduced routes.

Let it be further LM_i, $i = \overline{1,l}$ – vector of route lengths by number of nodes VM_i, $i = \overline{1,l}$ – vector of route numbers, ordered in descending order of the number of nodes in the route. In the process of route reduction, we will build a reduction vector VR_i, $i = \overline{1,l}$ and reference vector RV_i, $i = \overline{1,l}$. The elements of vectors are defined as follows:

$$VR_i = \begin{cases} 0, & \text{if the route } i \text{ - th is not reduced to any other rout,} \\ k, & \text{if the route } i \text{ - th is reduced to some route } \xi, \end{cases}$$

where k – route number that has been reduced to a route ξ just before the route i. If i – the first reduced route, then $k = \xi$;

$$RV_i = \begin{cases} 0, & \text{if the route } i \text{ is reduced,} \\ \xi, & \text{if any route is reduced to a route } i, \\ & \text{but it itself is not reduced to any other route,} \\ i, & \text{if the route } i \text{ itself is not reduced to any other} \\ & \text{and no route is reduced to it,} \end{cases}$$

where ξ – the number of the last route reduced to a route i.

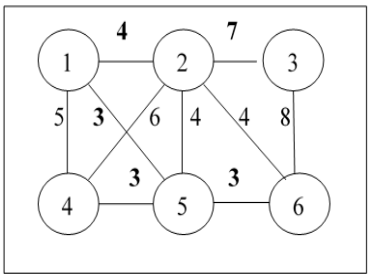
Each element of the vector $VR_i = 0$ corresponding to the route m_i determines a chain of other routes reduced to this route. The ordering of routes in a chain is the opposite of the order of their reduction. The first route in the chain for VR_i specifies the element RV_i. As a result of the reduction algorithm given in [4], the chains in the reduction vector for $VR_i = 0$, the beginning of which is indicated by RV_i, will be ordered by increasing number of nodes in the reduced routes.

After executing the reduction algorithm, a reference structure is built. At the same time, only such routes are visible, in which $VR_i = 0$. Obviously, after executing the reduction algorithm, the amount of RAM of the computer occupied by the reference structure H_S will be reduced by the amount of: $\Delta = \left(\sum_{k=1}^{l} \eta_k (\eta_k - 1)/2 - \sum_{\alpha \in \{k^*\}} \eta_\alpha (\eta_\alpha - 1)/2 \right) V_E$, where V_E is the amount of memory occupied by one item in the list. The time complexity of reduction algorithm by nodes is on average $O(l^2(n + \eta_{cp}) + l\eta_{cp})$, and in arcs – $O(l^2 \eta_{cp})$ operations, where η_{cp} – average route length. Route reduction is extremely necessary if the initial set of routes is all possible routes. In this case, the dimensionality of the problem increases sharply, taking on the character of building new, previously non-existent routes.

Algorithm for the formation of a reference structure H_S with the simultaneous construction of route arcs of the network G_M is also given in [4]. Medium complexity of the algorithm for the formation of the reference structure H_S is $O((l - lr)\eta_{cp}^2)$ operations, where lr – the number of reduced routes, and η_{cp} – average route length. For the initial distribution of flows on the network G_M an algorithm is used to build the shortest paths according to the criterion: minimum transit overloads; if the number of overloads is equal – the minimum length of the path [5].

III. An Example of the Formation of a Reference Structure

Let a given network with the number of nodes $n = 6$, number of arcs $p = 10$ and a matrix of arc lengths R, which is shown in Fig. 2. After reducing the specified routes by nodes (U) and arcs (A) the results shown in Fig. 3, 4.



R						
N	1	2	3	4	5	6
1	**0**	**4**	**0**	**5**	**3**	**0**
2	**4**	**0**	**7**	**6**	**4**	**4**
3	**0**	**7**	**0**	**0**	**0**	**8**
4	**5**	**6**	**0**	**0**	**3**	**0**
5	**3**	**4**	**0**	**3**	**0**	**3**
6	**0**	**4**	**8**	**0**	**3**	**0**

Fig. 2. Specified network and arc length matrix

Options	Reduction	
	By nodes U	By arcs A
Number of nodes in the network	6	6
Number of arcs in the network	10	10
Number of routes in the network	14	14
Number of reduced routes, incl. in %	13, 92%	6, 42%
Number of elements in the ADT structure	72	72
Number of reduced elements, incl. in %	57, 79%	17, 23%

Fig. 3. Reduction results

№	$S=\|\| s_{ij} \|\|_{14\times6}$						LM_{14}	VM_{14}	VR_{14}	RV_{14}	VR_{14}	RV_{14}
									U		A	
1	1	2	3	6	5	4	6	1	0	6	0	13
2	1	2	3	6	0	0	4	9	4	0	1	0
3	1	2	6	5	0	0	4	3	9	0	0	3
4	1	2	5	6	0	0	4	4	3	0	0	10
5	1	2	4	0	0	0	3	2	14	0	0	5
6	1	4	0	0	0	0	2	12	13	0	8	0
7	2	4	5	0	0	0	3	14	11	0	9	0
8	2	4	1	0	0	0	3	5	7	0	0	6
9	2	4	5	6	3	0	5	10	1	0	0	7
10	2	5	6	0	0	0	3	11	5	0	4	0
11	3	2	1	0	0	0	3	7	10	0	2	0
12	3	2	5	6	0	0	4	8	2	0	0	12
13	3	6	0	0	0	0	2	13	8	0	11	0
14	4	5	2	6	0	0	4	6	12	0	0	14

Fig. 4. The results are obtained after the reduction of the specified routes by nodes (U)and arcs (A)

Fig. 5. excerpts from the reference structure H_S for flows u_{12} and u_{21} without route reduction and with reduction by nodes and arcs.

When distributing flows in the first case, the routes will be visible in the following order – 8, 11, 5, 2, 4, 3, 1; in the second – 6, 13, 8, 7, 11, 10, 5, 14, 12, 2, 4, 3, 9, 1; in the third – 8, 6, 5, 4, 10, 3, 1, 13, 11, 2, 1.Thus, as already noted, after the reduction of routes in the main optimization algorithm, the total number of route views increases, which accordingly leads to an increase in its labor intensity. In addition, in the case of arc reduction, the revision of routes by increasing the number of nodes may be disrupted if the same flow can pass along paths with different arcs.In our example, these are both flows – u_{12} and u_{21} (routes 1, 2, 3, 4, 5, 11 and route 8).

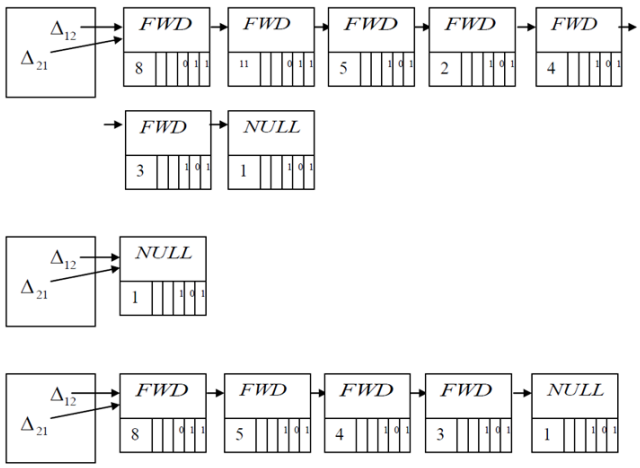


Fig. 5. Fragments of the reference structure H_S for flows u_{12} and u_{21} without route reduction and with reduction by nodes and arcs

Fig. 6 shows a matrix of shortest path lengths D and reference matrix of built paths $C=\|\, c_{ij} \,\|_{n\times n}$, each element of which c_{ij}, $i\neq j$ determines the number of the penultimate node on the shortest path from i before j, $c_{ii}=0$, $i=\overline{1,n}$. Matrices obtained for the network G_M algorithm [5].

D						
N	1	2	3	4	5	6
1	**0**	**4**	**11**	**5**	**8**	**8**
2	**4**	**0**	**7**	**6**	**4**	**4**
3	**11**	**7**	**0**	**14**	**11**	**8**
4	**5**	**6**	**14**	**0**	**3**	**6**
5	**8**	**4**	**11**	**3**	**0**	**3**
6	**8**	**4**	**8**	**6**	**3**	**0**

C						
N	1	2	3	4	5	6
1	**0**	**1**	**1**	**1**	**1**	**1**
2	**2**	**0**	**2**	**2**	**2**	**2**
3	**3**	**3**	**0**	**3**	**3**	**3**
4	**4**	**4**	**4**	**0**	**4**	**4**
5	**5**	**5**	**5**	**5**	**0**	**5**
6	**6**	**6**	**6**	**6**	**6**	**0**

Fig. 6. Shortest path length matrix D and reference matrix of built paths C

IV. Results of the Computational Experiment

To test the performance and computational efficiency of the algorithms, a computational experiment was conducted on a PC with a clock frequency of 2.66 GHz and 2 GB of RAM running the Windows 10 operating system. The algorithms were tested on networks containing 50 to 500 nodes. For the entire dimension, the pseudorandom number sensor generated homogeneous networks with the number of output arcs from each node (a measure of the nodes) val = 5 and topological arc lengths ranging from 80 to 320. The results of the calculations are given in Fig. 7-10. Where lr and $lr\%$, E, Er and $Er\%$, $TFLOYD$, $TRED,U$, $TRED,A$, $TFORM$, $TVVSP$ — respectively, the number and percentage of reduced routes; the number of elements generated and the number and percentage of reduced elements E in the structure H_S; time to build the shortest paths according to the criterion — the minimum length of the path on the network G Floyd's algorithm [8, 9] to generate routes for all n^2-n flows; time of reduction of routes by nodes and arcs; time of formation of the structure H_S; the time of constructing the shortest paths by the algorithm [10] according to the criterion – minimum transit overloads, if the number of overloads is equal – the minimum length of the path on the network G_M.

n	50	100	150	200	250	300	400	500
val	5	5	5	5	5	5	5	5
p	125	250	375	500	625	750	1000	1250
l	1225	4950	11175	19900	31125	44850	79800	124750
η_{max}	7	9	10	13	11	12	13	14
η_{cp}	3	4	5	5	6	6	6	6
lr $lr\%$	789 64%	3382 68%	7764 69%	14165 71%	22102 71%	31336 69%	55682 69%	88619 71%
E	7797	48961	138596	301079	510913	786289	1618257	2822650
Er $Er\%$	3904 50%	27196 55%	79606 57%	181025 60%	308204 60%	467208 59%	967801 59%	1743565 61%
TFLOYD	0.02	0.02	0.05	0.08	0.16	0.28	0.64	1.20
TRED.U *TRED.A*	0.02 0.00	0.28 0.16	2.12 0.61	6.32 1.81	17.88 4.35	53.38 9.47	184.81 31.47	546.85 85.47
TFORM	0.00	0.02	0.03	0.06	0.11	0.27	1.11	2.65
TVVSP	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00

Fig. 7. Results of a computational experiment on networks containing 50 to 500 nodes

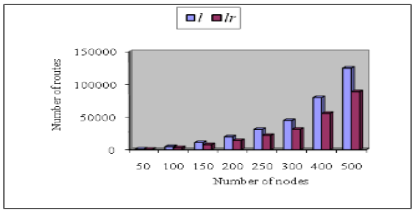


Fig. 8. Number of routes

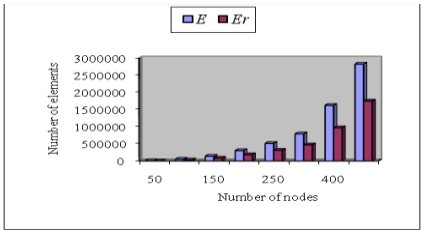


Fig. 9. Number of elements

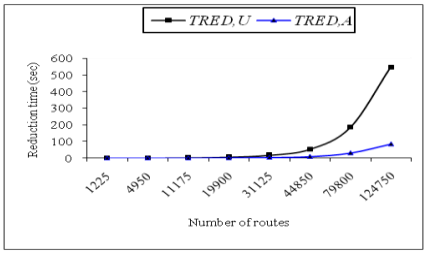


Fig. 10. Reduction time (sec)

V. Conclusion

1. The paper develops data structures and algorithms for route reduction, which allow solving the problem of distribution and routing of flows in the case when the number of specified routes is too many, and the amount of RAM of the computer is limited. Data structures based on linear dynamic lists are proposed, with the help of which you can quickly find all possible routes for distributing flows in the network.

Estimates of the labor intensity of algorithms for reducing routes by nodes have been obtained – $O(l^2(n+\eta_{cp})+l\eta_{cp})$ and arcs – $O(l^2\eta_{cp})$, as well as an algorithm for the formation of a reference data structure – $O((l-lr)\eta_{cp}^2)$, where l and n – the number of routes (the number of specified communication lines) and nodes in the network, η_{cp} – average route length (number of nodes in the communication line), lr – number of reduced routes (number of reduced communication lines). The proposed algorithms were tested on networks with the number of nodes from 50 to 500 and the number of routes from 1225 to 124750, which showed their efficiency and good computational efficiency, and they can be used in practical problems of distribution and routing of flows on large-dimensional networks.

2. On the basis of the proposed data structures and heuristic approaches, polynomial algorithms and a computer program have been developed [6] for solving the NP-hard [7] problem of distribution and routing of flows of transport blocks. Estimates of the temporal complexity of algorithms are obtained. Algorithms allow you to get an approximate solution to the problem in time $T_o = O(n^3) + O(l^2(n+\eta_{cp})+l\eta_{cp})$ (or $O(l^2\eta_{cp}) + O((l-lr)\eta_{cp}^2) + \gamma\,[\,O(n_g n^{4+\varepsilon}) + O[\lambda(e(e+\alpha)+Ke(e+\alpha)) + e\log e + K_1 e]\,]$,

where γ – number of iterations of solving the problem of selecting arc capacities [8], n_g – the average number of parts in a branched flow, e – the number of arcs in the network, α – the number of different discrete bandwidths of the network arcs, λ, K, K_1 – some constants, $\varepsilon \in [0, 0.3]$.

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[8] O.M. Trofymchuk and V.A. Vasyanin, "Choosing the Capacity of Arcs with Constraint on Flow Delay Time," Cybernetics and Systems Analysis. 2019. 55(4). pp. 561-569.
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