Using Zadeh's Implication for Approximate Reasoning

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***Abstract*— In z-set based modelling, the inputs of the discussed control systems can represent Z-values. It is clear that If-Then rules, in linguistic describtions with input and output parts, are characterized with Z-set based information. In existing literature it characterize probabilistic and fuzzy uncertainty. There are a set of method for fuzzy reasoning approach and a probabilistic approximate reasoning approach, also all this methods use separately with fuzzy uncertainty and probabilistic uncertainty. A new method is needed to overcome this gap. For this reason we try to develop a new approach for approximate reasoning. In this research, we propose a unique method for approximate inference utilizing Z-set-based principles. A computational illustration includes four Z-set-based guidelines for approximate reasoning.**

***Keywords*— Z-set, Z-rule, fuzzy implication, Zadeh's implication, Z-If-Then rules Introduction**

I Approximate inference is a research field in artificial intelligence (AI) that relies on human judgment and has been extensively studied for decades.

In the scientific literature there are a set of reasoning approaches related on inac-curacy information. If uncertainty in a research is a stochastic (random) one, then knowledge is presented and reasoning is performed using probability theory. If un-certainty in a study arises from the inherent imprecision of information, then fuzzy reasoning serves as a suitable method for application. We can declare that the first research on approximate reasoning was carried out by Zadeh [1-5]. But today in scientific literature, there are new Z- set based reasoning methods[6,7] that they use Zadeh's Z-number theory. Zadeh suggested that the main advantage of human rea-soning. He developed a detailed model for fuzzy reasoning in approximate terms [8-10].

Zadeh's proposed method [11] introduces a new framework for handling uncer-tainty. This method encompasses probability theory and predicate logic, enabling the management of various forms of uncertainty.

Zadeh [10] introduced the notion of a linguistic variable and its use in approxi-mate reasoning. In his famous 3-paper research, Zadeh wrote: "probabilities, too, can be treated as linguistic variables with values such as likely, very likely, unlikely, etc. Computation with linguistic probabilities requires the solution of nonlinear pro-grams and leads to results which are imprecise to the same degree as the underlying probabilities. The main applications of the linguistic approach lie in the realm of humanistic systems – especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas". It is clear that probability theory does not offer a solution for merging two probability measures. Because it is a problem of judgment, but it isn't of mathematics.

Currently, Z-numbers and Z-sets serve as powerful tools for representing imperfect information [12-14].

In this paper, we will apply a novel method for approximate reasoning using Z-set-based rules and Zadeh's implication.

I. Description of the Basic Definitions

Definition 1[6]. Z If-Then rules.

If X_1 is $Z_{X_1,n}=(A_{X_1,n},B_{X_1,n})$ and,…, and X_m is $Z_{X_m,n}=(A_{X_m,n},B_{X_m,n})$ then Y is $Z_Y=(A_{Y,n},B_{Y,n})$

$Z_{X_m,n}=(A_{X_m,n},B_{X_m,n})$ is the Z-sets based inputs and $Z_Y=(A_{Y,n},B_{Y,n})$ is Z-sets based outputs of the rules.

Definition 2. Z-sets[6]. Let X be a space of objects and $x\in X$. A Z-set Z in X is defined by membership function $A(x)\in[0,1]$, probability distributions P_X associated with X, membership function $B(P(x))\in[0,1]$; where $A(x)$ denotes that element x belongs to A with membership degree $A(x)$, $B(P(x))$ denotes that probability measure value belongs to B with degree $B(P(x))$, P_X is the underlying probability density of X. There is a restriction $P_X=\int_X A(x)p(x)dx$.

Considering that the degree to which P_X satisfies Z-set $\{\text{Prob}\}_P(A(x),\forall x\in X)$ is B, we can get $G_P=B(\int_X A(x)p(x)dx)$.

So Z-set in X can be expressed as triple $Z=(A,B,G)$, where A is fuzzy subset of X, B is fuzzy set (restriction) on probability measure of A, G is set of probability distribution over the space P.

$P(A)$, induced by a set of distributions G:

$$G=\left\{p_Z(x):\int_X p_Z(x)dx=1,\int_X p_Z(x)A(x)dx \text{ is } B, x\in X\right\}. \quad (1)$$

Definition 3[7]. Aggregation probability distributions (probability aggregation method performed by computing the weighted sum):

$$P_G(p) = \sum_{i-1}^{n} w_i P_i(p).$$

In the equation, W_i are positive weighting factors: in order to have a meaningful global probability, their sum must be equal to 1.

Definition 4[7].Fuzzy composition.

$$\mu_U(u) = \max_u \min[\mu_{\mathrm{E}}(e_{ic}^c), \mu_{\mathrm{R}}(u,e)]$$

Where $\mu_{\mathrm{R}}(u,e)$ is the membership function of fuzzy relation, $\mu_{\mathrm{E}}(e_{ic}^c)$ - membership function of current input and $\mu_U(u)$ -membership function of the output.

III. STATEMENT OF THE PROBLEM

The problem to be solved in this paper can be presented as follows. There is a Z-set based inference system with n rules substituting its rulebase. The goal is to derive and present an adequate inference method, efficiently applicable in Z-set based systems.

Z information based If-Then rules is given below[7]:

If X_1 is $Z_{X_1,1} = (A_{X_1,1}, B_{X_1,1})$ and,…, and X_m is $Z_{X_m,1} = (A_{X_m,1}, B_{X_m,1})$ then Y is $Z_Y = (A_{Y,1}, B_{Y,1})$

If X_1 is $Z_{X_1,2} = (A_{X_1,2}, B_{X_1,2})$ and,…, and X_m is $Z_{X_m,2} = (A_{X_m,2}, B_{X_m,2})$ then Y is $Z_Y = (A_{Y,2}, B_{Y,2})$

…

If X_1 is $Z_{X_1,n} = (A_{X_1,n}, B_{X_1,n})$ and,…, and X_m is $Z_{X_m,n} = (A_{X_m,n}, B_{X_m,n})$ then Y is $Z_Y = (A_{Y,n}, B_{Y,n})$

and a current observation

X_1 is $Z'_{X_1} = (A'_{X_1}, B'_{X_1})$ and,…, and X_m is $Z'_{X_m} = (A'_{X_m}, B'_{X_m})$,

find the Z-value of Y.

Here m is the number of Z-valued input variables and n is the number of rules.

IV.Solution method

Solution method of the given problem consists of the following steps:

Step 1. Calculation of the A'' using Zadeh's fuzzy implication This step is to define for each rule a fuzzy relation matrix R_s, $\mathrm{s} = 1...4$, $R_s \subseteq \widetilde{E}_i \times \widetilde{U}_j$

The membership function $\mu_s(u,e)$ of the corresponding relation R_s has the form according to Zadeh implication:

$$\mu_s(u,e) = \max(\min(\mu_{\mathrm{E_i}}(e), \mu_{\mathrm{U_j}})1 - \mu_{\mathrm{E_i}}(e))$$

$$\mathrm{i,j,s} = \overline{1,n} \text{ (n=4)}$$

Step 2. Formulate union of fuzzy relation matrix R : $R = \cup R_S, s = 1,\ldots,4$

Step 3. Calculate the membership function using the compositional rule of inference,

$$\mu_U(u) = \max_u \min[\mu_{\mathrm{E}}(e_{ic}^c), \mu_{\mathrm{R}}(u,e)]$$

Step 4.Calculation B: Defining a set of probabilty distributions G using points of support B over inputs of the rules.

Step 5. Aggregation of the obtained distributions using the a weighted sum probability aggregation method :

$$P_G(p) = \sum_{i-1}^{n} w_i P_i(p).$$

In the eqution W_i are weighting factors.

Step 6. Defining a set of probabilty distributions G using points of support B over outputs of the rules.

Step 7. Calculaton a minimum values for every rule using the distribution of the inputs and outputs.

Step 8.Obtaining set of probability distributions using aggregation method.

Step 9. Calculation the set of probability distribution using points of support over new input.

Step 10. The distance between two vectors (the new input-based probability vector and the aggregated probability vector over the inputs of the rules) is calculated using the Hellinger distance [2]:

$$ZHD(P_{Z_1}, P_{Z_2}) = \min_{p_{zi} \in G} \left(\frac{1}{\sqrt{2}} \left(\sum_{j=1}^{k} (p_{z1_j}^{1/2} - p_{z2_j}^{1/2}) \right)^{1/2} \right)$$

Step 11. Calculation similarity measure[7]: $SM = 1/(1 + ZHD)$

Step 12. Calculation support points of output B an membership value of output B using A part of Z-set, similarity measure and probability distributions.

V.Numerical example

Z-number based inference

Let's assume that the following four rules are part of the fuzzy control system: [7].

1. If the error *e* is [negative big (NB),small sure] THEN the control action *u* is [negative big (NB), small sure].
2. If the error *e* is[negative medium (NM), sure] THEN the control action *u* is [negative medium (NM),sure].
3. If the error *e* is [negative small (NS), very sure] THEN the control action *u* is [negative small(NS),very sure].
4. If the error *e* is [zero (ZE),medium] THEN the control action *u* is [zero,medium].

Here, the errors in the fuzzy control system are represented by e, and the control actions are denoted as u. The membership function can be computed through the following rules:

$$small\ sure = 0.3/0.7 + 1/0.73 + 0.2/0.75$$
$$medium = 0.2/0.5 + 1/0.6 + 0.4/0.8$$
$$sure = 0.1/0.7 + 1/0.8 + 0.3/0.9$$
$$very\ sure = 0.2/0.8 + 1/0.9 + 0.2/1$$

Rule 1:

$$\mu_{E1}(e) = 1.00/-10 + 0.73/-7 + 0.34/-3 + 0.20/0 + 0.13/3 + 0.08/7 + 0.06/10$$
$$\mu_{U1}(u) = 1.00/-1 + 0.92/-0.7 + 0.67/-0.3 + 0.50/0 + 0.37/0.3 + 0.26/0.7 + 0.20/1$$

Rule 2:

$$\mu_{E2}(e) = 0.73/-10 + 1/-7 + 0.61/-3 + 0.34/0 + 0.20/3 + 0.11/7 + 0.08/10$$
$$\mu_{U2}(u) = 0.91/-1 + 1.00/-0.7 + 0.86/-0.3 + 0.67/0 + 0.50/0.3 + 0.34/0.7 + 0.26/1$$

Rule 3:

$$\mu_{E3}(e) = 0.34/-10 + 0.61/-7 + 1.00/-3 + 0.73/0 + 0.41/3 + 0.20/7 + 0.13/10$$
$$\mu_{U3}(u) = 0.61/-1 + 0.86/-0.7 + 1.00/-0.3 + 0.92/0 + 0.74/0.3 + 0.50/0.7 + 0.37/1$$

Rule 4:

$$\mu_{E4}(e) = 0.20/-10 + 0.34/-7 + 0.73/-3 + 1.00/0 + 0.73/3 + 0.34/7 + 0.20/10$$
$$\mu_{U4}(u) = 0.50/-1 + 0.67/-0.7 + 0.92/-0.3 + 1.00/0 + 0.92/0.3 + 0.6726/0.7 + 0.50/1$$

For given 4 rules and their two properties (errors and control actions) membership functions were evaluated. Problem is to perform Z inference. New input:

$$(\mu_{\mathrm{E}}(e^{c}_{ic}) = 0.11/-10 + 0.17/-7 + 0.34/-3 + 0.61/0 + 0.96/3 + 0.73/7 + 0.41/10, sure)$$

Let us consider the results step by step on proposed method.
Step 1. Construction fuzzy matrices on base of fuzzy implication Zadeh. The results of the first step is given in Table 1 and 4 .

Table 1. Fuzzy relation matrix R1

	1	0.92	0.67	0.5	0.37	0.26	0.2
1	1	0,92	0,67	0,5	0,37	0,26	0,2
0.73	0,73	0,73	0,67	0,5	0,37	0,26	0,2
0.34	0,34	0,34	0,34	0,34	0,34	0,26	0,2
0.2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
0.13	0,13	0,13	0,13	0,13	0,13	0,13	0,13
0.08	0,08	0,08	0,08	0,08	0,08	0,08	0,08
0.06	0,06	0,06	0,06	0,06	0,06	0,06	0,06

Table 2. Fuzzy relation matrix R2

0,73	0,73	0,73	0,73	0,67	0,5	0,34	0,27
1	0,91	1	0,86	0,67	0,5	0,34	0,26
0,61	0,61	0,61	0,61	0,61	0,5	0,39	0,39
0,34	0,66	0,66	0,66	0,66	0,66	0,66	0,66
0,2	0,8	0,8	0,8	0,8	0,8	0,8	0,8
0,11	0,89	0,89	0,89	0,89	0,89	0,89	0,89
0,08	0,92	0,92	0,92	0,92	0,92	0,92	0,92
0,73	0,73	0,73	0,73	0,67	0,5	0,34	0,27

Table 3. Fuzzy relation matrix R3

	0,61	0,86	1	0,92	0,74	0,5	0,37
0,34	0,66	0,66	0,66	0,66	0,66	0,66	0,66
0,61	0,61	0,61	0,61	0,61	0,61	0,5	0,39
1	0,61	0,86	1	0,92	0,74	0,5	0,37
0,73	0,61	0,73	0,73	0,73	0,73	0,5	0,37
0,41	0,59	0,59	0,59	0,59	0,59	0,59	0,59
0,2	0,8	0,8	0,8	0,8	0,8	0,8	0,8
0,13	0,87	0,87	0,87	0,87	0,87	0,87	0,87

Table 4. Fuzzy relation matrix R4

	0,5	0,67	0,92	1	0,92	0,67	0,5
0,2	0,8	0,8	0,8	0,8	0,8	0,8	0,8
0,34	0,66	0,66	0,66	0,66	0,66	0,66	0,66
0,73	0,5	0,67	0,73	0,73	0,73	0,67	0,5
1	0,5	0,67	0,92	1	0,92	0,67	0,5
0,73	0,5	0,67	0,73	0,73	0,73	0,67	0,5
0,34	0,66	0,66	0,66	0,66	0,66	0,66	0,66
0,2	0,8	0,8	0,8	0,8	0,8	0,8	0,8

Step 2. Defined a composed fuzzy relation matrix R is described below:

Table 5. Composed fuzzy relation

1	0,92	0,8	0,8	0,8	0,8	0,8
0,91	1	0,86	0,67	0,66	0,66	0,66
0,61	0,86	1	0,92	0,74	0,67	0,5
0,66	0,73	0,92	1	0,92	0,67	0,66
0,8	0,8	0,8	0,8	0,8	0,8	0,8
0,89	0,89	0,89	0,89	0,89	0,89	0,89
0,92	0,92	0,92	0,92	0,92	0,92	0,92

Step 3.The membership function of the control action defined using the compositional rule of inference.

If fuzzy input is represented the following form

$$\mu_{Enew}(e) = 0.11/-10 + 0.17/-7 + 0.34/-3 + 0.61/0 + 0.96/3 + 0.73/7 + 0.41/10$$

Then obtained output as follow:

$$\mu_{Eout}(e) = 0.61/-1 + 0.73/-0.7 + 0.89/-0.3 + 0.89/0 + 0.89/0.89 + 0.89/0.7 + 0.89/1$$

Step 4. The calculation of the probability distribution for all antecedents of all rules is as follows:

For instance, a fragment of the probability distributions on the antecedents of the rules, obtained according to Definition 1, is provided in Table 6.

Table 6. Probability distributions on the rule antecedents

On the second rule (input)			…	On the fourth rule (input)		
b21=0,7	**b22=0,8**	**b23=0,9**	…	***B41=0.4***	***B42=0.6***	***B43=0.8***
0,272857	0,056212	0	…	0,137943	0,051722	0
0,365441	0,601298	0,855719	…	0,155969	0,16725	0,030795
0,17603	0,245022	0	…	0,160447	0,203154	0,294047
0,043281	0,001565	0	…	0,161615	0,226079	0,375452
0,006483	0	0	…	0,11021	0,152917	0,25006
0,037003	0	0	…	0,038749	0,05003	0,049646
0,098905	0,095904	0,144281	…	0,235068	0,148848	0

Step 5.Obtained distribution of all rules as shown in Table 7.

Table 7. Aggregation of probability distribution

P'1	P'2	P'3
0,133381681	0,072692	0,098375984
0,157654205	0,177215	0,12236236
0,308843024	0,359928	0,377230637
0,16331118	0,20228	0,18772607
0,082913019	0,076458	0,125029934
0,023999666	0,025015	0,024823188
0,129897349	0,086412	0,064452075
1,000000125	1	1,00000025

Result of the Step 4 is given below:

Table 8. Obtained probability distribution over consequent part of the rule

distributions (on output of the second rule)			…	distributions (on output of the fourth rule)		
0,7	0,8	0,9	**…**	***0.5***	***0.6***	***0.8***
0,460520957	0,030918	0	…	0	0.208884	0.21
0,068071665	0,316043	0	…	0.060591261	0	0
0,049007157	0,287351	0,97594	…	0.140805025	0	0
0,076627034	0,196512	0	…	0.174276175	0	0
0,094383799	0,121984	0	…	0.19101044	0	0
0,085182411	0,047193	0	…	0.203559604	0.791116	0.79
0,166207976	0	0,02406	…	0.229758495	0	0

The resulting set of probability distributions, using support points B over the new input (Table 9), is shown as follows:

Table 9. Probability vector over the new input

New input distribution (P)		
0.7	***0.8***	***0.9***
0	0	0
0.0425751	-1E-06	-1E-06
0.11625711	0.0386041	0.0
0.1994741	0.2749131	0.1548211
0.3344691	0.5132371	0.8199081
0.2640721	0.1732471	0.0252721
0.0431521	0.0	0.0

Calculated distance between probability vector of the new input and aggregated probability vector over inputs of the rule as follow:

$$ZHD(P_{Z_1}, P_{Z_2}) = 0{,}251$$

Similarity measure has calculated using distance and obtained measure is given below: SM=0.799361.
Obtained result:

$$A = 0.61/-1 + 0.73/-0.7 + 0.89/-0.3 + 0.89/0 + 0.89/0.89 + 0.89/0.7 + 0.89/1$$
$$B = 0.1/0.4817137 + 0.1/0.4817138 + 0.1/0.4817139 + 1/0{,}5221568 +$$
$$+ 0{,}2/0{,}5221570 + 0{,}2/0{,}5677165 + 0{,}2/0{,}5677165 + 0{,}1/0{,}5777834$$

Result of computer simulation is described as follows(Fig. 1).

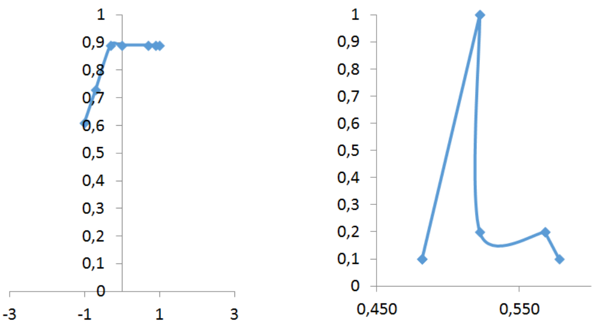

Fig. 1. Result of the computer simulation

VI.Conclusion

In this paper, we apply the reasoning method is described in [7] using Zadeh impli-cation. Authors of the paper[7] have used Z-set based reasoning using Ali-3 impli-caton and copula. We have developed analyzing the behavior of the control system using the implication and Z set presented by Zadeh. For simplicity, the minimum operation was used instead of the copula.The results obtained by us are the results of calculations on the algorithm, which we discussed in the article.The advantage of this method is the ability to process information characterized by fuzzy and proba-bilistic characteristics together. It should be noted that existing methods can process this type of information separately. In the considered algorithm, Zadeh implication, distance concept, relevance measure, fuzzy and probabilistic aggregation were used. This approach is possible to process imperfect information and can be used in deci-sion-making in various fields.

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